Workshop on weak-coupling expansion for the pressure of hot QCD

4 loops 3-d SU(3) free energy with a mass IR regulator

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Why 4 loops 3-d SU(3) free energy?

The free energy is related to the mean value of the trace of the pl

$$\langle Tr(1-\Pi_p)\rangle = \frac{c_1}{\beta_0} + \frac{c_2}{\beta_0^2} + \frac{c_3}{\beta_0^3} + \frac{\tilde{c}_4}{\beta_0^4} + O(\beta_0^{-5})$$

The first three coefficients have already been computed(*) while th

is ***IR divergent***

Need for a regulat

- Coupling constant
- Finite volume
- Mass

Consistent with continuum computations

(*) F. Di Renzo, A. Mantovi, Y Schröder, V. Miccio, *JHEP* **0405** (2004) 006

Without repeating the conceptual and computational frame of this work, let's directly merge into its core, that is the ***partition function*** of the theory :

$$Z = \int [D\phi] det\left(-\sum_\mu \hat{\partial}^L_\mu \hat{D}_\mu(\phi) + m^2\right) e^{-S_W - S_{GF}} = \int [D\phi] e^{-S_W - S_{GF} - S_{FP}}$$

where

$$S_W = \beta_0 \sum_p Tr(1 - \Pi_p) + \frac{\beta_0 m^2}{12} \sum_{n,\mu,A} \phi^A_\mu(n) \phi^A_\mu(n)$$

Gluon mass term

$$S_{GF} = \frac{\beta_0}{12\alpha} \sum_{n,A} \left[\sum_\mu \hat{\partial}^L_\mu \phi^A_\mu(n)\right]^2$$

Gauge parameter

$$S_{FP} = -Tr\left[ln\left(-\sum_\mu \hat{\partial}^L_\mu \hat{D}_\mu[\phi] + m^2\right)\right]$$

Ghost mass term

with

$$\hat{D}_\mu(\phi) = \left[1 + \frac{i}{2}\phi_\mu(n) - \frac{1}{12}\phi^2_\mu(n) - \frac{1}{720}\phi^4_\mu(n) - \frac{1}{30240}\phi^6_\mu(n) + 0[\phi^8_\mu(n)]\right]\hat{\partial}^R_\mu + i\phi_\mu(n)$$

Because of ***Langevin equation***, we have to face the expression

$$\nabla S_{FP} = -\nabla Tr[lnB] = -Tr[\nabla B B^{-1}]$$

with

$$B[\phi] = -\sum_\mu \hat{\partial}^L_\mu \hat{D}_\mu[\phi] + m^2$$

whose treatment shows two interesting features:

1) B inversion

$$B^{-1} = [B_0]^{-1} + \\ -[B_0]^{-1} B_1 [B_0]^{-1} + \\ -[B_0]^{-1} B_2 [B_0]^{-1} - B_0^{-1} B_1 [B^{-1}]_1 + \dots$$

It's necessary to invert B_0 only!

2) Trace computation

$$\mathrm{Tr}[\nabla B B^{-1}] = \sum_{i,j} (\nabla B)_{ij} B_{ji} = \\ \sum_{j,k} \xi_i (\nabla B)_{ij} B_{jk} \xi_k$$

with

$$\langle \xi_i \xi_k \rangle = \delta_{i,k}$$

No explicit ghost fields!

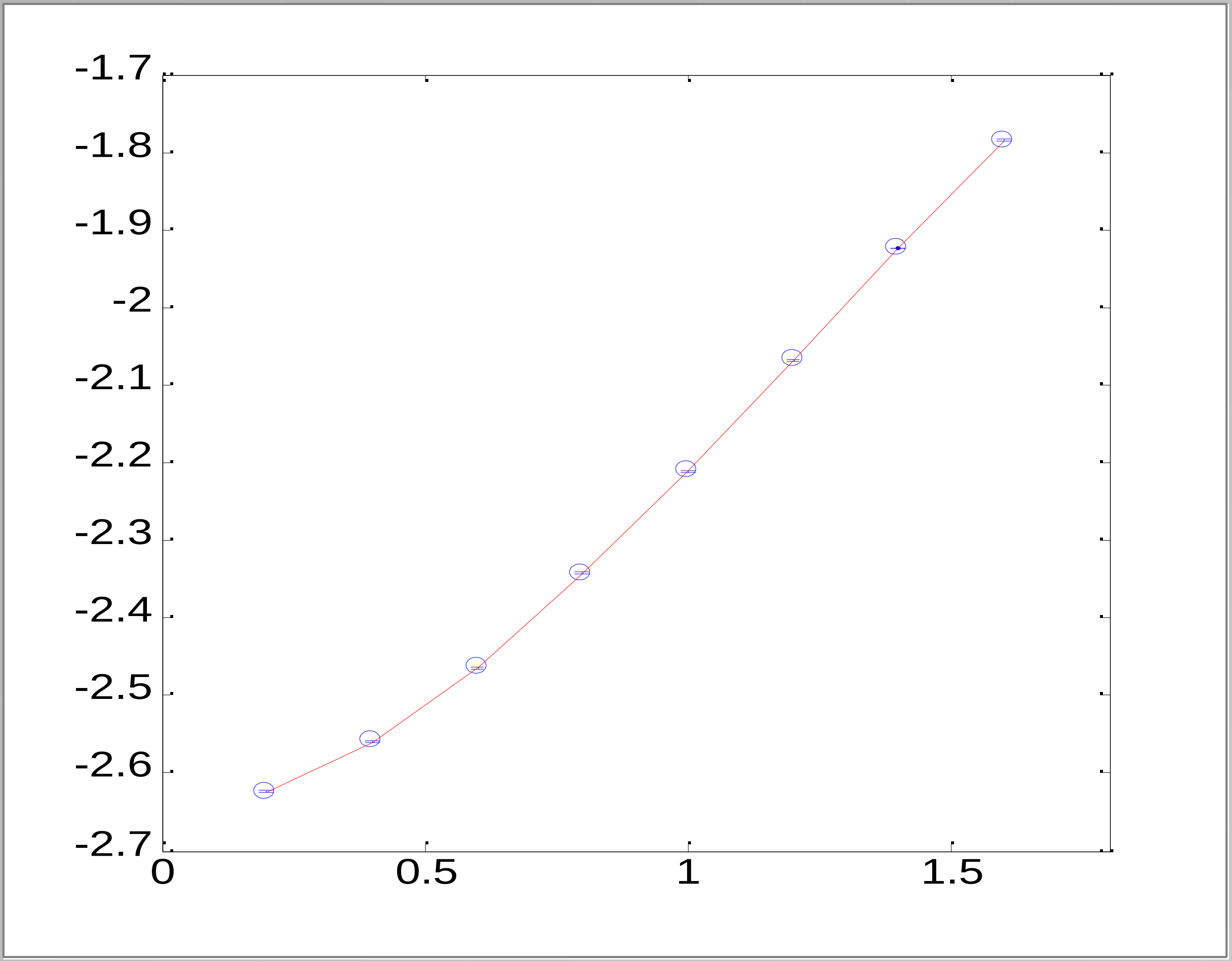
Let's resume the global strategy:

First you fix a lattice size and a mass value, perform simulations for different discretized stochastic time τ, take the average on the thermalized signals and then extrapolate to $\tau=0$

Repeat the procedure for different lattice sizes fitting the results to obtain the **<u>infinite volume</u>** value

Repeat the procedure for different masses, subtract the logarithmic divergence from the fourth loop and finally extrapolate to **<u>zero mass</u>**

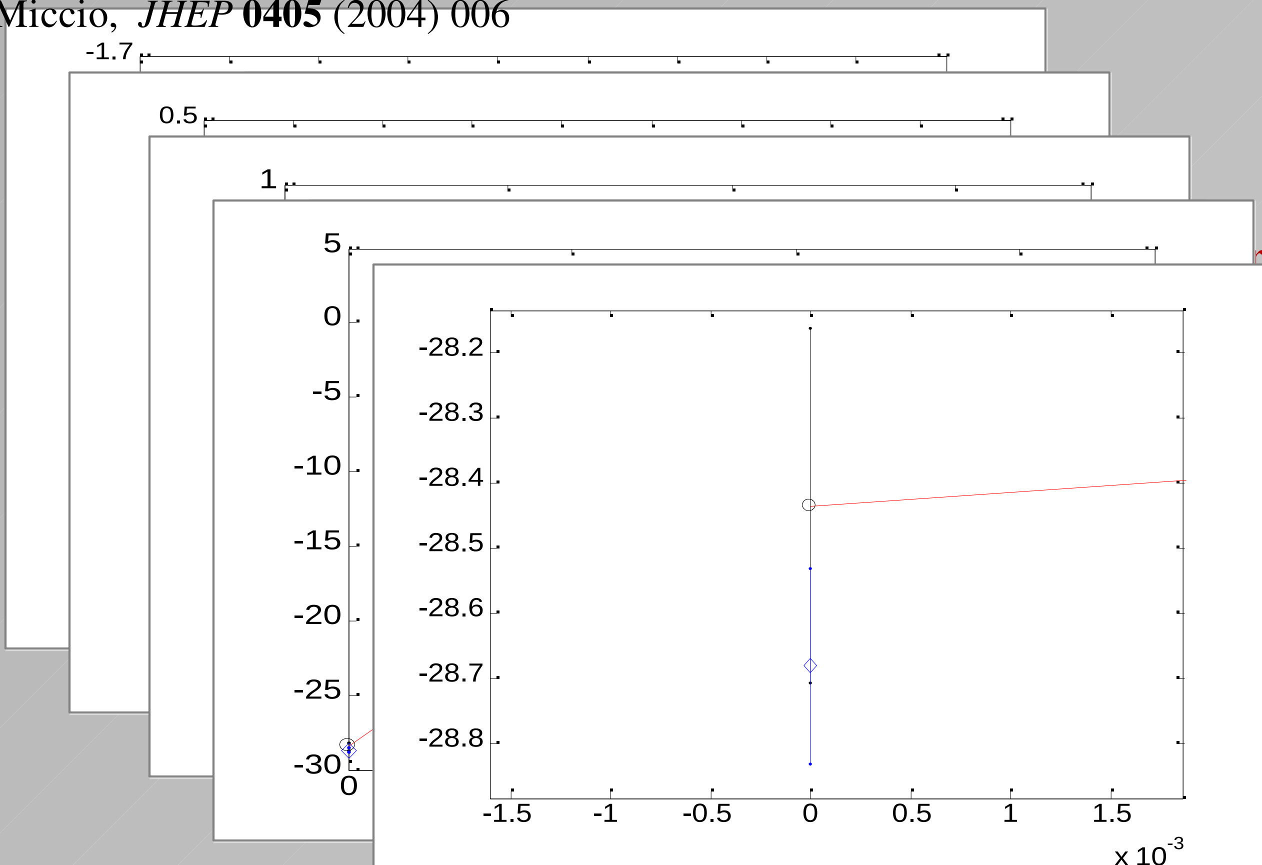
Reliability check I: comparison between numerical values and analytic ones at first loop for the plaquette.



M	An. value	Num. value
0.2	-2.6234	-2.6240(16)
0.4	-2.5605	-2.5583(15)
0.6	-2.4643	-2.4642(14)
0.8	-2.3442	-2.3424(12)
1.0	-2.2093	-2.2111(11)
1.2	-2.0674	-2.0677(11)
1.4	-1.9243	-1.9239(9)
1.6	-1.7842	-1.7847(8)

Reliability check II: comparison between zero mass extrapolated values and the expected ones.

F. Di Renzo, A. Mantovi, Y Schröder, V. Miccio, *JHEP* **0405** (2004) 006



Comparison of the third loo

values at different lattice sizes

rst loop (Lattice ze = 7)

Extrapolation

L	Exp. value	Ext. value
5	-5.990(26)	-6.001(15)
8	-6.408(16)	-6.392(14)
12	-6.591(11)	-6.595(36)
16	-6.658(9)	-6.641(30)

Let's take a reliability check of this last reliability check, that is let's see how stable are the coefficients of the various powers of the extrapolation

Masses	$m^0 coeff$	$m^2 coeff$	$m^4 coeff$	$m^6 coeff$	$m^8 coeff$	$m^{10} coeff$	$m^{12} coeff$
4	-6.153(28)	13.46	-9.27	-	-	-	-
5	-6.240(30)	15.19	-15.92	-6.50	-	-	-
6	-6.277(31)	16.05	-20.17	13.46	-3.52	-	-
7	-6.295(38)	16.55	-23.03	19.44	-8.64	1.51	-
8	-6.303(40)	16.78	-24.71	23.81	-13.69	4.15	-0.50

Expected value: -6.307(21)

The leading coefficients appear to be quite stable

Let's extrapolate to **infinite volume** using the first loop analytic values

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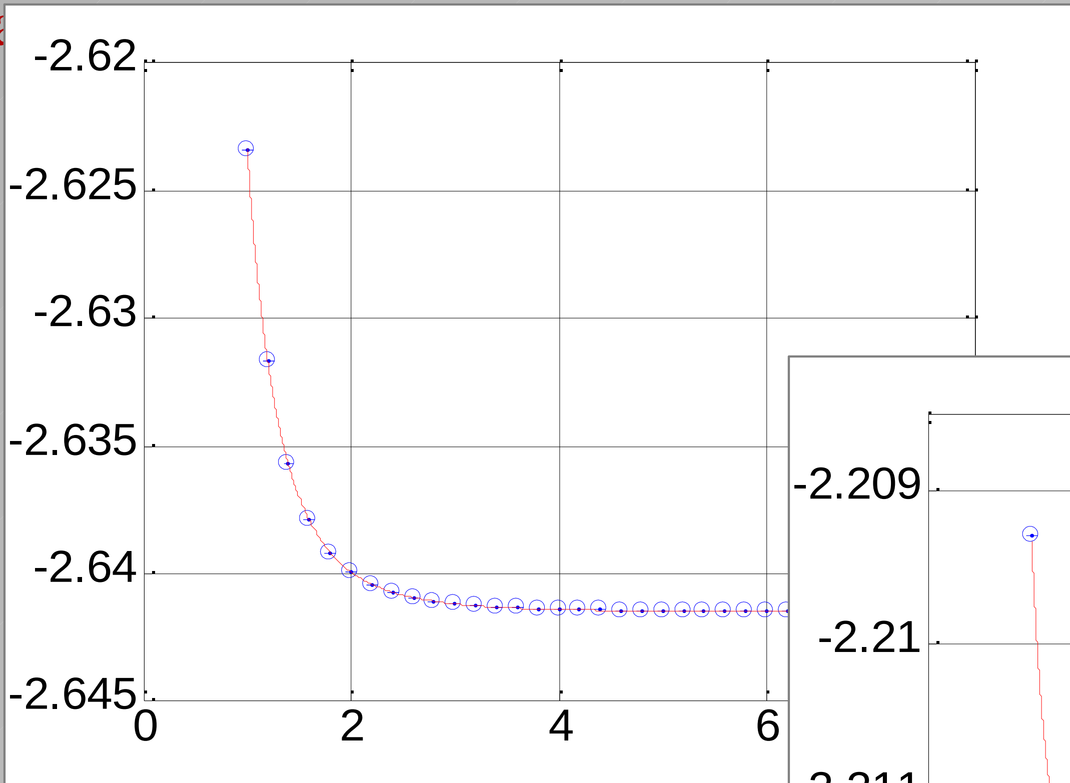
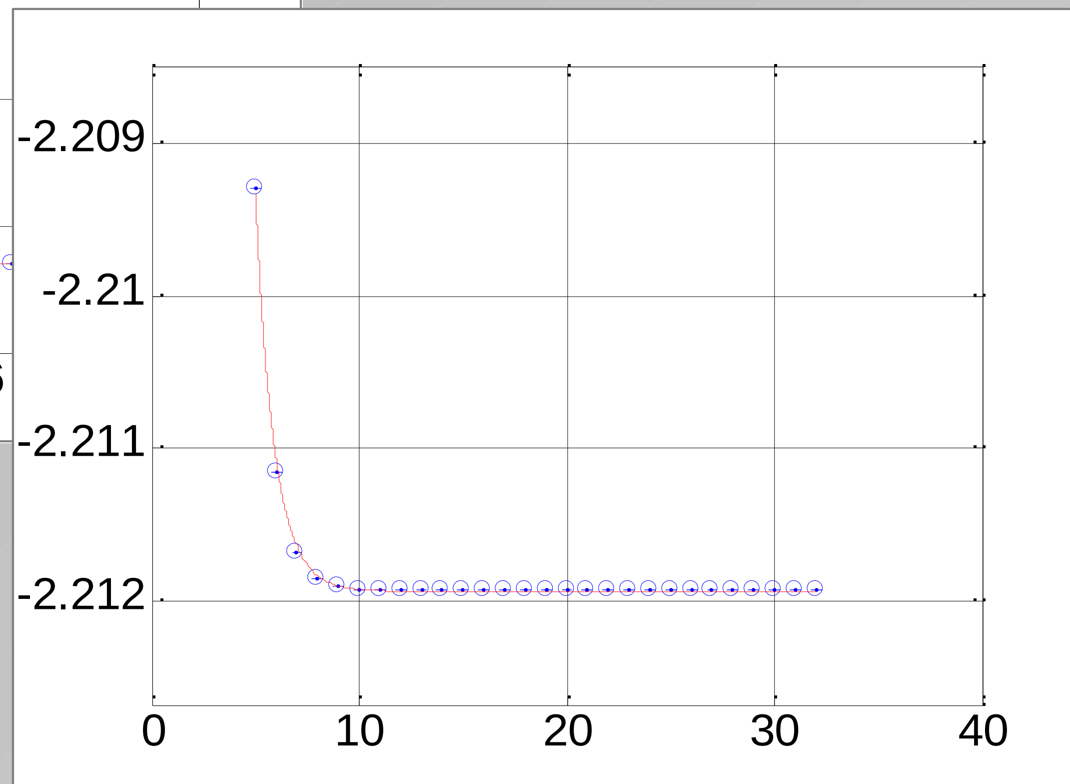


Fitting curve (m=0.2 , x=mL

$$y = c_1 + e^{-x}\left(c_2 + c_3 x^{-2} + c_4 x^{-4}\right)$$



Fitting curve (m=0.2 , x=mL)

$$y = c_1 + e^{-x}\left(c_2 + c_3 x^{-2} + c_4 x^{-4}\right)$$

As expected, the decay towards infinite volume is ***slower*** at small masses and this is confirmed by numerical data also for the ***fourth***
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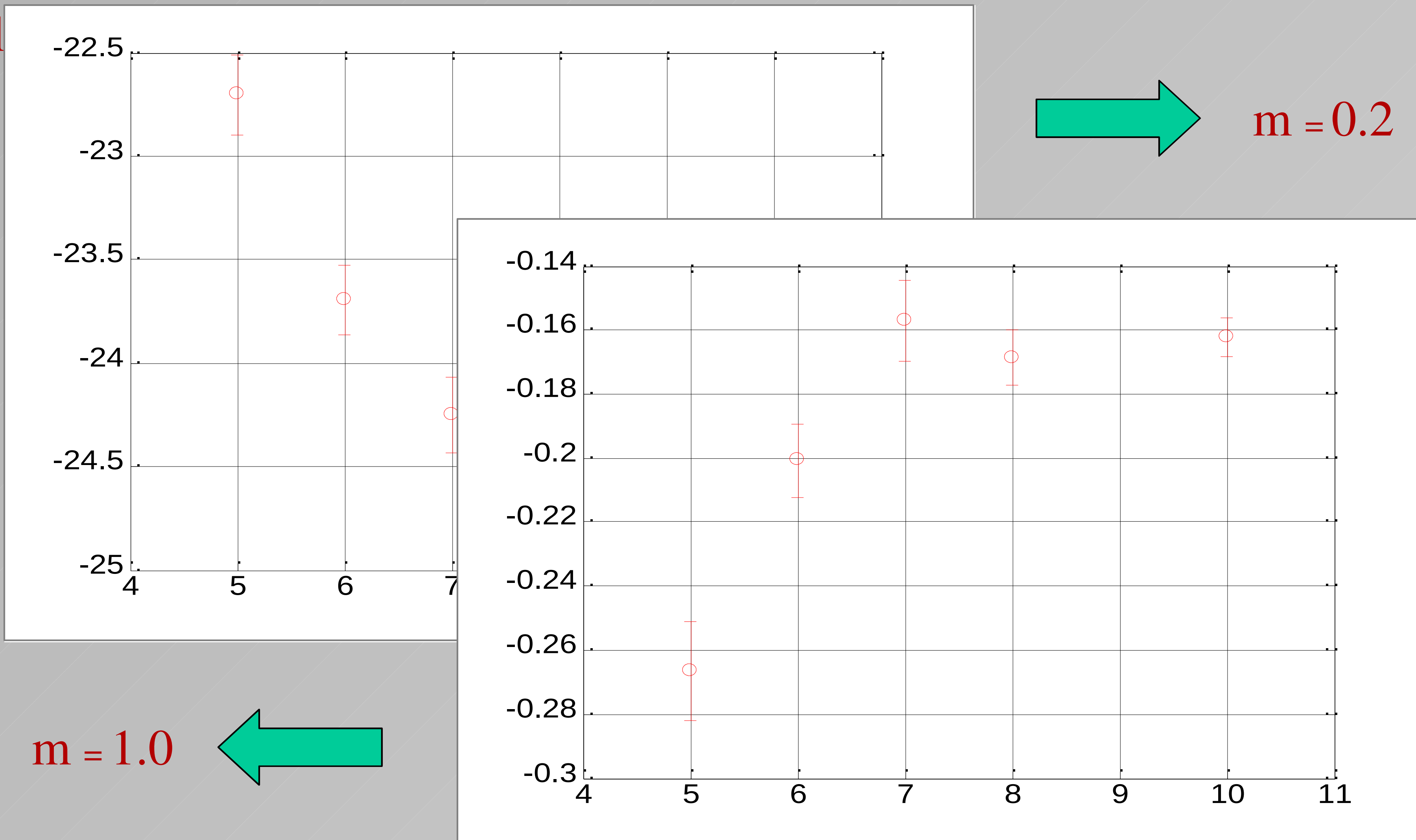

m = 0.2

m = 1.0

Conclusions and perspects

First results appear encouraging since they behave as expected

Extrapolation to infinite volume seems to be tougher at small masses but it will be possible to get precise values by means of bigger lattices

Subsequent remotion of IR divergence and extrapolation to zero mass appear feasible with good accuracy

The same algorithm could be used to determine any observable at fixed covariant gauge