Lattice measurements of non-perturbative condensates in EQCD

Ari Hietanen

in collabration with K. Kajantie, M. Lainke, K. Rummukainen and Y.Schröder

September 8-10, 2005, Trento, Italy

Introduction

- We study the 3d electrostatic QCD (EQCD), which is defined as follows:

$$\mathcal{S}_{\mathrm{E}} = \int \mathrm{d}^d x \mathcal{L}_{\mathrm{E}} \quad (1)$$

$$\mathcal{L}_{\mathrm{E}} = \frac{1}{2}\mathrm{Tr}[F_{ij}^2] + \mathrm{Tr}[D_i, A_0]^2 + m_3^2 \mathrm{Tr}[A_0^2] + \lambda_3 (\mathrm{Tr}[A_0^2])^2. \quad (2)$$

- It can be matched to full QCD using perturbation theory.
- The perturbative series for the pressure of QCD converges very slowly.
- The reason for this can be traced back to the slow convergence of the perturbative series for $\partial_{m_3^2} f_{\overline{\mathrm{MS}}}$ inside 3d EQCD.
- The perturbative expansion of pressure is know up to $g^6 \ln g$. The g^6 term can be calculated using 3d pure Yang-Mills theory (MQCD).
- Lattice measurements in EQCD can give an estimate of the sum of all higher order terms, starting at the level g^7.

Outline of the procedure

We can not measure the free energy (or pressure) directly from lattice. However we can calculate derivatives of it (called condensates).

The analysis can be divided into following steps:

I Measure the condensates on lattice as a function of lattice spacing a.

II Subtract the difference of $\overline{\mathrm{MS}}$ and lattice regularisation schemes and extrapolate the results to continuum.

IV Subtract the known perturbative $\overline{\mathrm{MS}}$ result and fit the higher order terms.

V Integrate the result to obtain the the contribution to free energy. The integration constant (g^6-term) comes from MQCD.

Continuum definitions

- The bare vacuum energy density of EQCD is defined as

$$f \equiv \lim_{V\to\infty} \ln \int \mathcal{D}A_k \mathcal{D}A_0 \exp(-S_E). \qquad (3)$$

- We also define the dimensionless ratios

$$\begin{aligned} x &\equiv \frac{\lambda_3}{g_3^2} \\ y &\equiv \frac{m_2^2}{g_3^4} \\ \mathcal{F}(x,y) &\equiv \frac{f}{g_3^6}. \end{aligned} \qquad (4)$$

- x and y are related as follows:

$$y = \frac{(9-N_f)(6+N_f)}{144\pi^2 x} + \frac{486 - 33N_f - 11N_f^2 - 2N_f^3}{96(9-N_f)\pi^2} + \mathcal{O}(x). \qquad (5)$$

- For $N_f = 0$ in leading order $x \approx 0.04/y$.

Lattice definitions

- Using the standard Wilson discretization we get the lattice action

$$\begin{aligned} S_a &= S_a^{YM} + S_a^{AH} \\ S_a^{YM} &= \beta \sum_{x} \sum_{i<j} \left(1 - \frac{1}{N_c} \mathrm{ReTr}[P_{ij}(\vec{x})]\right) \\ S_a^{AH} &= 2a \sum_{\vec{x}} \sum_{i} \left\{ \mathrm{Tr}[A_0^2(\vec{x})] - \mathrm{Tr}[A_0(\vec{x}) U_i(\vec{x}) A_0(\vec{x}+\imath) U_i^{\dagger}(\vec{x})] \right\} + \\ &\quad + a^3 \sum_{\vec{x}} \left\{ m_{\mathrm{bare}}^2 \mathrm{Tr}[A_0^2(\vec{x})] + \lambda_3 (\mathrm{Tr}[A_0^2(\vec{x})])^2 \right\}, \end{aligned} \quad (6)$$

where

$$\beta \equiv \frac{2N_c}{g_3^2 a}. \quad (7)$$

Relating the $\overline{\text{MS}}$ and lattice free energies

- First we define $\overline{\text{MS}}$

$$f_{\overline{\text{MS}}} \equiv \lim_{\epsilon \to 0} [f - f_{\text{div}}], \tag{8}$$

where f_{dif} is divergent part of f conataining all the poles ϵ and all the factors $\mu^{-2\epsilon}$.

- For dimensional reasons, $f_{\overline{\text{MS}}}(m_3^2(\bar{\mu}), g_3^2, \lambda_3; \bar{\mu})$ can be parametrised as

$$\begin{aligned} f_{\overline{\text{MS}}} &= \frac{d_a}{4\pi}[B_{1,0}]m_3^3 \\ &+ \frac{d_a}{(4\pi)^2}[B_{2,0}g_3^2 + B_{2,1}\lambda_3]m_3^2 + \\ &+ \frac{d_a}{(4\pi)^3}[B_{3,0}g_3^4 + B_{3,1}g_3^2\lambda_3 + B_{3,2}\lambda_3^2]m_3 + \\ &+ \frac{d_a}{(4\pi)^4}[B_{4,0}g_3^6 + B_{4,1}g_3^4\lambda_3 + B_{4,2}g_3^2\lambda_3^2 + B_{4,3}\lambda_3^3] + \end{aligned} \tag{9}$$

- To match the lattice result to $f_{\overline{\rm MS}}$ we need the difference between lattice and $\overline{\rm MS}$ schemes. Because the 3d theory is super-renormalisable, there are divergent contributions only to four loop order. Consequently it can be parametrized as

$$\begin{aligned}
\Delta f &\equiv f_a(m_3^2(\bar{\mu}), g_3^2, \lambda_3; \bar{\mu}) - f_{\overline{\rm MS}}(m_3^2(\bar{\mu}), g_3^2, \lambda_3; \bar{\mu}) \\
&= \frac{d_a}{4\pi}\left[C_{1,0}\frac{1}{a^3} + D_{1,0}\frac{m_3^2(\bar{\mu})}{a}\right] \\
&+ \frac{d_a}{(4\pi)^2}\left[C_{2,0}\frac{g_3^2}{a^2} + C_{2,1}\frac{\lambda_3}{a^2} + D_{2,0}g_3^2 m_3 + D_{2,1}\lambda_3 m_3^2\right] \\
&+ \frac{d_a}{(4\pi)^3}\left[C_{3,0}\frac{g_3^4}{a} + C_{3,1}\frac{g_3^2\lambda_3}{a} + C_{3,2}\frac{\lambda_3^2}{a}\right] \\
&+ \frac{d_a}{(4\pi)^4}\left[C_{4,0}g_3^6 + C_{4,1}g_3^4\lambda_3 + C_{4,2}g_3^2\lambda_3^2 + C_{4,3}\lambda_3^3\right] + \\
&+ \mathcal{O}(a).
\end{aligned} \tag{10}$$

The condensate $\langle \mathrm{Tr}[A_0^2] \rangle$

- The derivative of the vacuum energy dencity with respect to mass parameter defines the quadratic condensate,

$$\partial_{m_3^2(\bar{\mu})} f_a^{AH} = \langle \mathrm{Tr}[A_0^2] \rangle_a. \tag{11}$$

- This can be matched to $\overline{\mathrm{MS}}$

$$\begin{aligned}\partial_y \mathcal{F}_{\overline{\mathrm{MS}}}^{AH}(x, y; \mu) &= \lim_{\beta\to\infty} \left\{ \langle \mathrm{Tr}[A_0^2] \rangle_a - \partial_y \frac{\Delta f}{g_3^6} \right\} \\ &= \lim_{\beta\to\infty} \left\{ \langle \mathrm{Tr}[A_0^2] \rangle_a - \left[\tilde{c}_1 \beta + \tilde{c}_2 (\ln \beta + \tilde{c}_2{}') \right] \right\}, \end{aligned} \tag{12}$$

  where $\tilde{c}_1$, $\tilde{c}_2$ and $\tilde{c}_2{}'$ are numerical constants.

- The scale has been set to $\bar{\mu} = g_3^2$.

- Perturbatively the quadratic condensate is as follows:

$$\partial_y \mathcal{F}_{\overline{\mathrm{MS}}}^{AH}(x,y;\mu) = -0.32 y^{\frac{1}{2}} + y^0(0.038 + x) + y^{-\frac{1}{2}}(0.074 + x + x^2) + \\ + y^{-\frac{1}{2}}(0.074 + x + x^2) + y^{-1}(0.019 + x + x^2 + x^3) + \\ + y^{-\frac{3}{2}}(? + x + x^2 + x^3 + x^4) + \ldots \quad (13)$$

- Aim is to measure the $\partial_y \mathcal{F}_{\overline{\mathrm{MS}}}^{AH}(x,y;\mu)$ numerically as a function of y, such that we can fit the term order of $y^{-3/2}$.
- For each y the continuum limit must be taken.
- In general the expansion of $\partial_y \Delta f / g_3^6$ in $1/\beta$ is as follows:

$$\partial_y \frac{\Delta f}{g_3^6} = \tilde{c}_1 \beta + \tilde{c}_2(\ln \beta + \tilde{c}_2{}') + (\tilde{c}_3 \ln \beta + \tilde{c}_3{}'(y)) \frac{1}{\beta} \\ + (\tilde{c}_4{}'(y) \ln \beta + \tilde{c}_4(y)) \frac{1}{\beta^2} + \ldots \quad (14)$$

- The problem arises due to the logarithmic $\tilde{c}_3$ term.

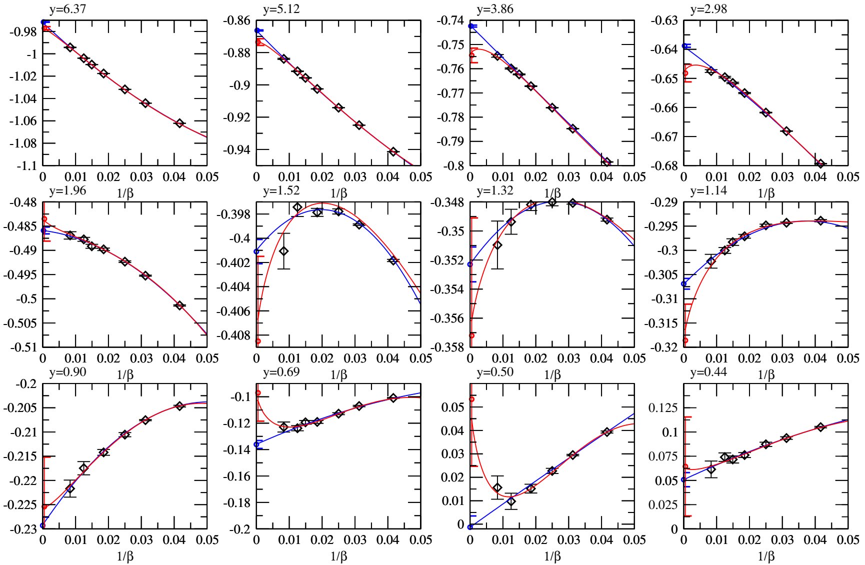


Figure: Continuum extrapolation of $\langle \mathrm{Tr}[A_0^2] \rangle$ after the subtraction of the divergent terms. Blue curve is a fit $B_{\mathrm{quad}} + \tilde{c}_3/\beta + \tilde{c}_4/\beta^2$ and red curve is a fit $B_{\mathrm{quad}} + (\tilde{c}_3 + \tilde{c}_3{}' \ln \beta)/\beta + \tilde{c}_4/\beta^2$

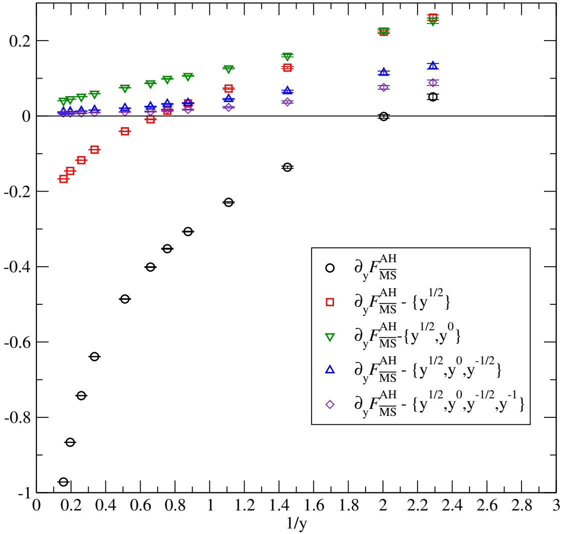


Figure: Subtraction of perturbative results order by order using non-logarithmic continuum extrapolation.

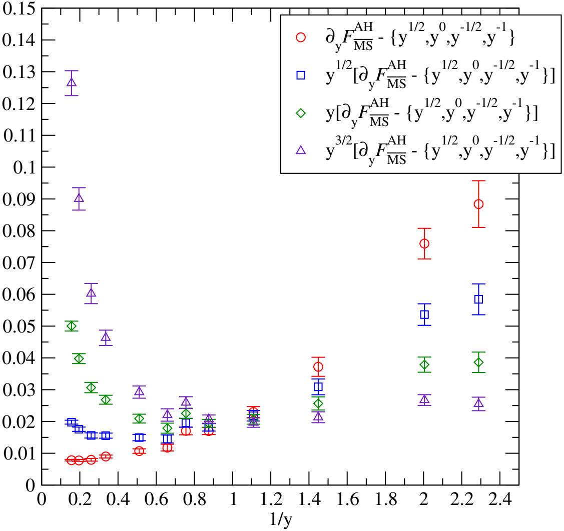


Figure: The continuum extrapolation after subtrating the known perturbative result should behave as $\propto y^{-3/2}$. However, the behavior is more like $\propto y^{-1/2}$.

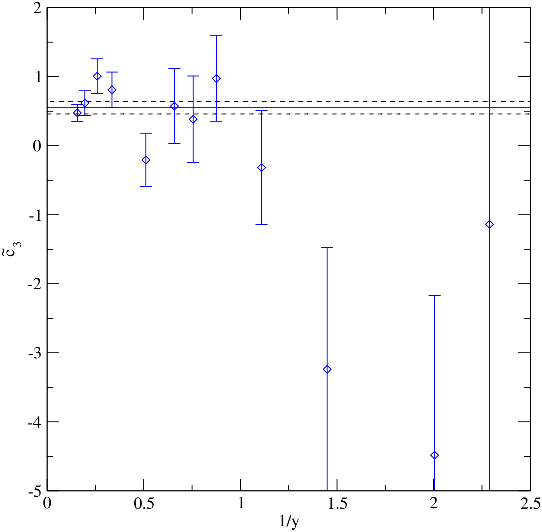


Figure: The term $\tilde{c}_3$ should bee independent from y, so we can fit it. The fit gives $\tilde{c}_3 = 0.55(8)$ with $\chi^2/\mathrm{dof} = 19.8/11$.

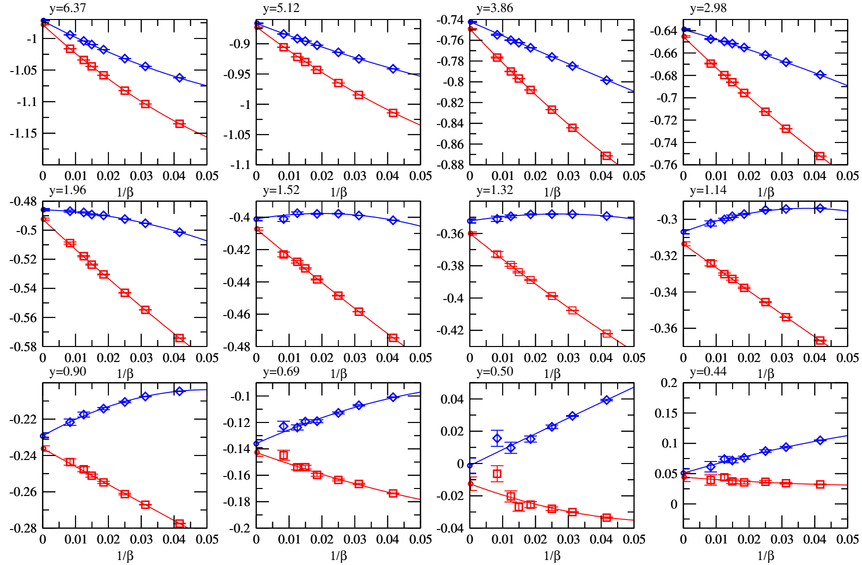


Figure: The blue curves are lattice data minus divergent terms. The red curves are data minus divergent terms minus $0.55 \ln \beta/\beta$.

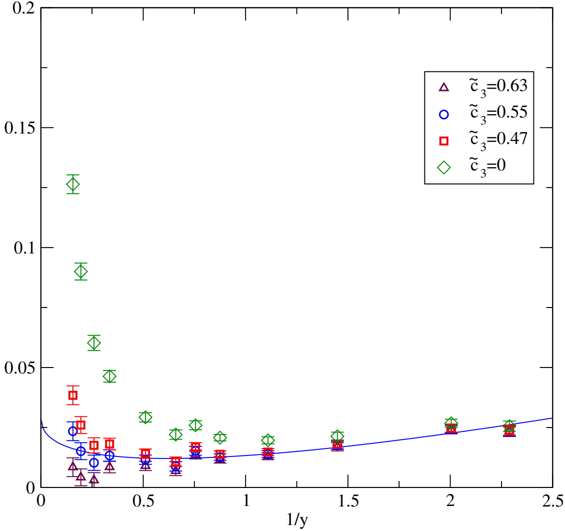


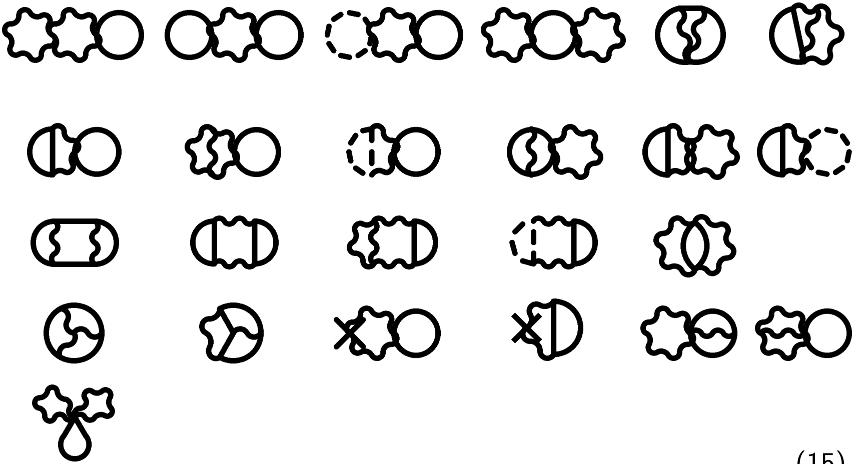
Figure: Value of $y^{\frac{3}{2}}(\partial_y \mathcal{F}^{AH}_{\overline{\mathrm{MS}}} - \{y^{\frac{1}{2}}, y^0, y^{-\frac{1}{2}}, y^{-1}\})$ with different values of $\tilde{c}_3$. Blue curve is a fit for $\tilde{c}_3 = 0.55$ giving $0.029(6) - 0.042(11)y^{-\frac{1}{2}} + 0.027(6)y$ with $\chi^2/\mathrm{dof} = 13.1/9$.

- The fitted values are extremely sensitive for the value of $\tilde{c}_3$:

$\tilde{c}_3$	fit	χ^2/dof
0.47	$0.052(5) - 0.082(11)y^{-1/2} + 0.043(6)y^{-1}$	22.9/9
0.55	$0.029(6) - 0.042(11)y^{-1/2} + 0.027(6)y^{-1}$	13.1/9
0.63	$0.0048(60) - 0.002(10)y^{-1/2} + 0.011(5)y^{-1}$	9.2/9

- We should calculate the exact value of $\tilde{c}_3$. This can be done in lattice perturbation theory.
- We need terms $\mathcal{O}(am_3^2 g_3^4)$. They come from 3-loop Feynman diagrams including at least one massive line and with no pure λ vertex calculated to $\mathcal{O}(a)$.

- 3-loop diagrams $\sim g_3^4$ in EQCD.



(15)

- 3-loop diagrams $\sim g_3^2 \lambda$

(16)

- 3-loop diagrams $\sim \lambda^2$.

(17)

The condensate $\langle (\mathrm{Tr} A_0^2)^2 \rangle$

- The derivative of the vacuum energy dencity with respect to self-coupling λ_3 defines the quartic condensate,

$$\partial_{\lambda_3^2(\bar{\mu})} f_a^{AH} = \langle (\mathrm{Tr}[A_0^2])^2 \rangle_a + [\partial_{\lambda_3} \delta m_a^2] \langle \mathrm{Tr}[A_0^2] \rangle_a - \partial_{\lambda_3} f^{\mathrm{AH}}. \qquad (18)$$

- The matching to $\overline{\mathrm{MS}}$ leads to:

$$\begin{aligned}
\partial_x \mathcal{F}^{AH}_{\overline{\mathrm{MS}}}(x, y; \mu) = \lim_{\beta \to \infty} \Big\{ & \langle (\mathrm{Tr}[A_0^2/g_3^2])^2 \rangle_a & (19)\\
& - \left[\bar{c}_1 \beta + \bar{c}_2 (\ln \beta + \bar{c}_2') + x \bar{c}_3 (\ln \beta + \bar{c}_3') \right] \langle \mathrm{Tr}[A_0^2] \rangle_a \\
& - \Big[\hat{c}_1 \beta^2 + \hat{c}_2 \beta (\ln \beta + \bar{c}_2') + x \hat{c}_3 \beta (\ln \beta + \hat{c}_3{}') + \\
& + \quad (2x - N_c) \hat{c}_4 (\ln \beta + \bar{c}_2{}')^2 + \hat{c}_5 (\ln \beta + \hat{c}_5{}') + \\
& + \quad x \hat{c}_6 (\ln \beta + \hat{c}_6') + \quad x^2 \hat{c}_7 (\ln \beta + \hat{c}_7') \Big] \Big\}, & (20)
\end{aligned}$$

where $\bar{c}_i$ and $\hat{c}_i$ are numerical constant. $\hat{c}_5'$, $\hat{c}_6'$ and $\hat{c}_7'$ are still unknown.

- Pertubatively the quartic condensate is as follows:

$$\partial_x \mathcal{F}^{AH}_{\overline{\mathrm{MS}}} = y + y^{\frac{1}{2}}(1+x) + y^0(1+x+x^2) + y^{-\frac{1}{2}}(?+x+x^2+x^3) + \dots \tag{21}$$

- However we need to fit the unknown constant $\hat{c}_5'$. So after continuum extrapolations the result shuold behave as $\hat{c}_5' + y^{-1/2} + y^{-1} + \dots$
- We have to again do the continuum extrapolation for each y (this time there is no $\ln \beta/\beta$-term).

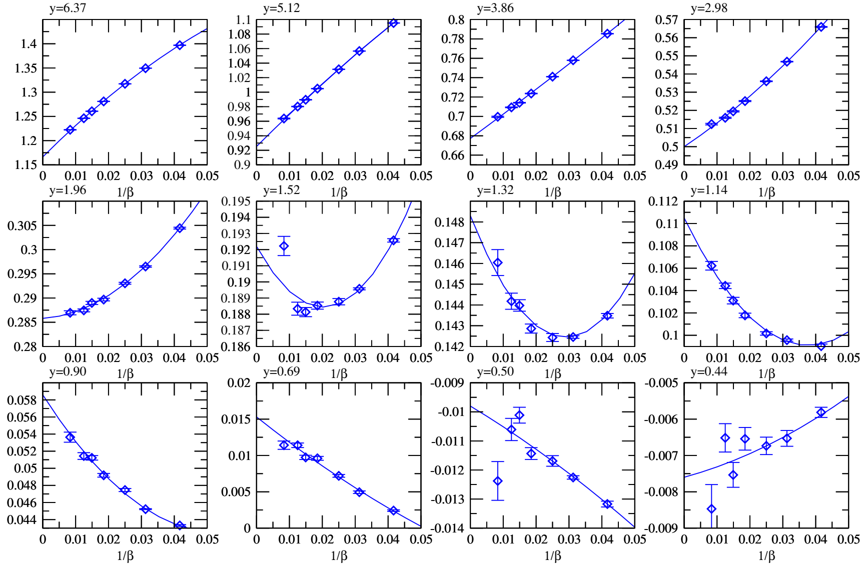


Figure: Continuum extrapolation of $\langle \mathrm{Tr}[A_0^4] \rangle$ after the subtraction of the divergent terms.

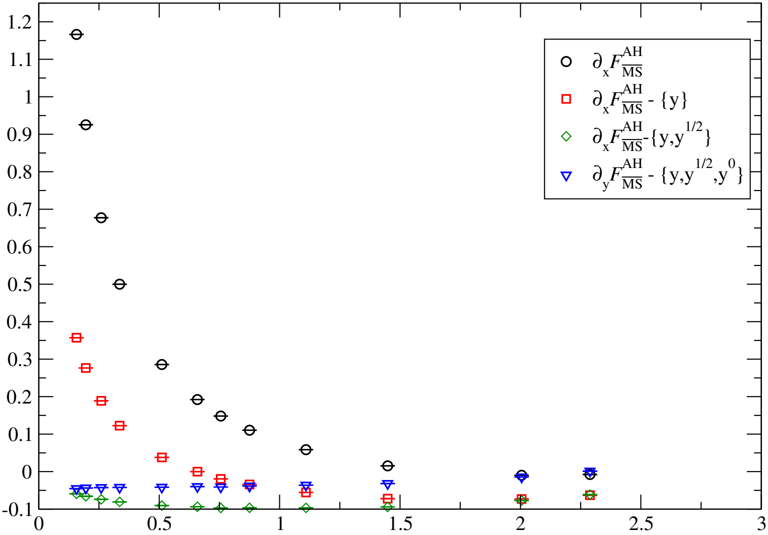


Figure: Subtraction of perturbative results order by order.

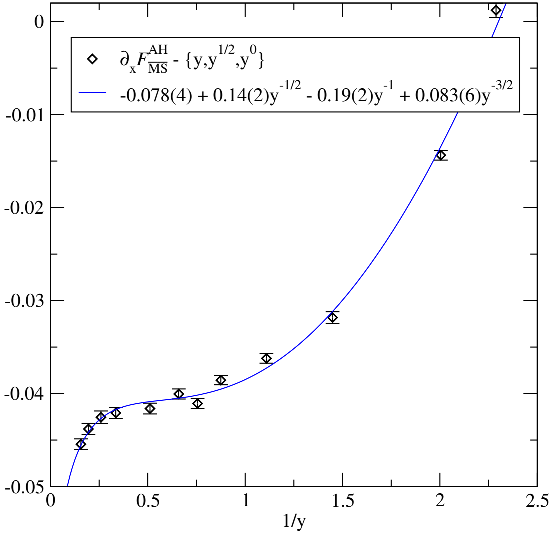


Figure: Fit of a third order polynomial to data with $\chi^2/\text{dof}= 23.5/8$. Possible systematic errors might cause problems

Summary

- From the condesate $\langle(\mathrm{Tr}[A_0^2])^2\rangle$ we can fit the unknown constant $\hat{c}_5'$ as well as order $xy^{-1/2}$ contribution to the pressure giving.

$$\hat{c}_5' = -0.078(4) \quad (22)$$

$$p_{\mathrm{QCD}} = \cdots + 0.14(2)xy^{-\frac{1}{2}} + \ldots . \quad (23)$$

- The measurements of condestate $\langle\mathrm{Tr}[A_0^2]\rangle$ contains still problems because of the unknown $\tilde{c}_3$.