(41)

<u>Zunächst:</u>

$$\mathcal{L}_I = -2\sqrt{2}\, G_F \sum_{D_L}\sum_{D_L'} \underbrace{\bar{D}^1_L \begin{pmatrix} 0 & 0 \\ \gamma^\mu & 0 \end{pmatrix} \hat{D}^1_L}_{D=\binom{1}{2},\ \bar{2}\gamma^\mu 1} \quad \underbrace{\bar{D}'^1_L \begin{pmatrix} 0 & \gamma^\mu \\ 0 & 0 \end{pmatrix} \hat{D}'^1_L}_{D_L'=\binom{3}{4},\ \bar{3}\gamma_\mu 4}$$

$$D_L, D_L' \in \{ Q'_{1L}, Q'_{2L}, L_{1L}, L_{2L} \}$$

Leptonen: $L_1 \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad L_2 \equiv \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{matrix} \leftarrow Q=0 \\ \leftarrow Q=-1 \end{matrix}$

Quarks: $Q_1' = \begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} \quad \begin{matrix} \leftarrow Q=+\frac{2}{3} \\ \leftarrow Q=-\frac{1}{3} \end{matrix}$

$$Q_2' = \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix} \quad \begin{matrix} \leftarrow Q=+\frac{2}{3} \\ \leftarrow Q=-\frac{1}{3} \end{matrix}$$

Cabibbo-angle $\theta_c \approx 13{,}1^\circ$

<u>Aus 17c:</u> $\Gamma = \frac{\rho_0}{8\pi m_k^2} |M|^2(\rho_0)\, \Theta(m_k - m_1 - m_2)$

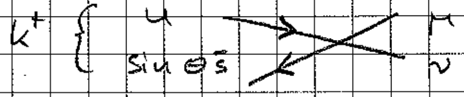
mit $\rho_0 \equiv \frac{1}{2m_k}\sqrt{(m_k \pm m_1 \pm m_2)(\ )(\ )}$

NR: $m_1 = m_2 \approx 0$: $\rho_0 = \frac{1}{2} m_k = 247\ \text{MeV}$

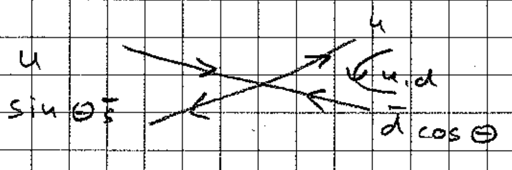
$k^+ \to \mu^+ \nu_\mu$ 63,5 % : $\rho_0 = \frac{1}{2}\sqrt{m_k^2 - 2m_\mu^2} = 235\ \text{MeV}$

$K^+ \to \pi^+ \pi^0$ 21,1 % : $\rho_0 = \frac{1}{2}\sqrt{m_k^2 - 4m_\pi^2} = 204\ \text{MeV}$

<u>Aus V-A Fermi:</u>



$M \sim G_F \sin\theta\, c$



$M \sim G_F \sin\theta \underbrace{\cos\theta}_{\approx 1} c$

$(\pi^+ = u\bar{d}\ ;\ \pi^0 = u\bar{u} - d\bar{d})$

<u>Aus dim. Analysis:</u> $\dim \Gamma = m$

$$\leadsto \dim |M|^2 \doteq m^2 = G_F^2\, m^6$$

$$\Rightarrow G_F^2 \rho_0^6 \Rightarrow c^2 \sim \rho_0^6$$

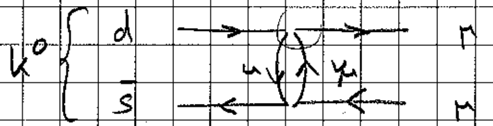
Insgesamt: $$\Gamma = \frac{\varrho_0}{8\pi m_K^2} \, G_F^2 \sin^2\theta_c \, \varrho_0^6$$

$$\tau = \frac{1}{\Gamma} = \frac{8\pi m_K^2}{G_F^2 \sin^2\theta_c \, \varrho_0^6} = \begin{pmatrix} 0{,}16 \\ 0{,}23 \\ 0{,}65 \end{pmatrix} \frac{10^8}{eV}$$

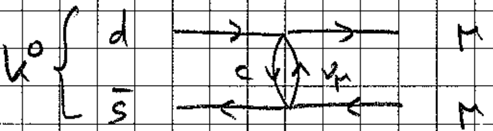
$$= \begin{pmatrix} 1{,}05 \\ 1{,}5 \\ 4{,}3 \end{pmatrix} 10^{-8}\,s$$

check PDB : $\underline{\underline{1{,}24 \cdot 10^{-8}\,s}}$

(42) Betrachte:



: use $Q_1' \rightarrow M \sim \begin{matrix} \cos\theta_c \\ \sin\theta_c \end{matrix}$



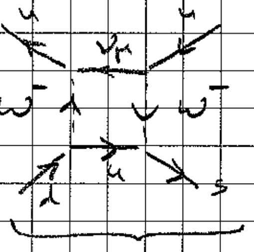
: use $Q_2' \rightarrow M \sim \begin{matrix} -\sin\theta_c \\ \cos\theta_c \end{matrix}$

$\Rightarrow$ Zwei der Kanäle kürzen sich in der Summe, da der Massenunterschied(u, c) nur im inneren Propagator $\frac{1}{p^2-m^2}$ steht.

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}$$

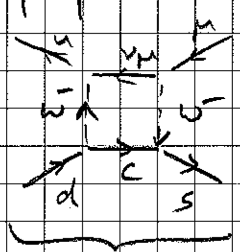
$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -d\sin\theta_c + s\cos\theta_c \end{pmatrix}$$

$$K^0 \rightarrow \mu^- \mu^+$$



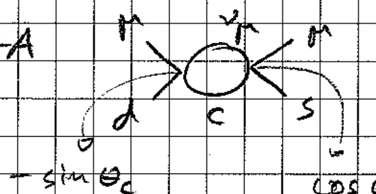
$K^0 = d\bar{s}$

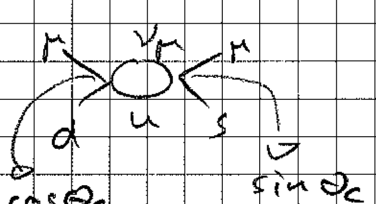
&



$K^0 = d\bar{s}$

in V-A $\rightarrow$





Amplitude: $\left.\begin{array}{l} M_1 \sim \cos\theta_c \sin\theta_c \\ M_2 \sim -\sin\theta_c \cos\theta_c \end{array}\right\} + = 0$

Bei Einbeziehung der Massendifferenz u, c heben sich die Amplituden nicht ganz weg.

(43)

<u>Kurze Variante:</u> $\Lambda \gg m \gg q_0, |\bar{q}|$

$$B \overset{Q=0}{\approx} \int \frac{1}{k^2 - m^2 + i\varepsilon} = \partial_{m^2} A \overset{m \ll \Lambda}{\approx} -\frac{i}{8\pi}\left(\ln\frac{m}{\Lambda} + \text{const}\right)$$

$$\text{da} \quad \partial_{m^2} A = \ln\frac{m}{\Lambda} + m^2 \, \partial_{m^2} \ln\frac{m}{\Lambda}$$

<u>Lange Variante:</u>

$$B = \int_0^\Lambda \frac{dr\, r^2}{2\pi} \int_{-1}^{1} \frac{d\cos\theta}{2\pi} \int_{-\infty}^{\infty} \frac{dr_0}{2\pi} \frac{1}{r_0 + \omega_1 - i\varepsilon} \frac{1}{r_0 - \omega_1 + i\varepsilon}$$

$$\cdot \frac{1}{r_0 + q_0 + \omega_2 - i\varepsilon} \frac{1}{r_0 + q_0 - \omega_2 + i\varepsilon}$$

wo $\omega_1 = \sqrt{\bar{r}^2 + m^2}$; $\omega_2 = \sqrt{(\bar{q} + \bar{r})^2 + m^2}$

$$= \sqrt{\bar{q}^2 + 2|\bar{q}|\, r \cos\theta + \omega_1^2}$$

- $\to$ aus Residuum nur 2 Terme
- $\to$ Angularterm $\to$ ln, arctanh
- $\to$ Radialterm $\to$ große $\sqrt{\ }$
- $\to$ Entwicklung "Series[Λ, Infinity, 0]"

$$\to \frac{i}{8\pi^2} \ln\frac{\Lambda}{m} + \text{fkt}\left(\frac{q_0}{m}, \frac{|\bar{q}|}{m}\right) + O\left(\frac{1}{\Lambda}\right)$$

Nutzung einer Mathematik Software, z.B. Mathematica, dringend empfohlen

(4)

$$A_\mu \rightarrow A_\mu' = A_\mu + \frac{1}{e}\partial_\mu \alpha$$

$$\phi(x) \longrightarrow \phi'(x) = e^{i\alpha(x)}\phi(x)$$

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi)$$

$$\mathcal{L}' = \left[\left(\partial_\mu - ie\left(A_\mu + \frac{1}{e}\partial_\mu \alpha\right)\right)\phi'\right]^\dagger$$
$$\left[\left(\partial^\mu - ie\left(A^\mu + \frac{1}{e}\partial^\mu \alpha\right)\right)\phi\right]$$

$$= \left[(\partial_\mu \phi)^\dagger e^{-i\alpha} + ie\phi^\dagger A_\mu^\dagger e^{i\alpha}\right]$$
$$\left[e^{i\alpha}\partial_\mu \phi - ieA^\mu e^{i\alpha}\phi\right]$$

$$= (\partial_\mu \phi)^\dagger (\partial^\mu \phi) + ie\phi^\dagger A^\dagger \partial_\mu \phi + ie\phi^\dagger A_\mu^\dagger \partial^\mu \phi$$
$$+ e^2 \phi^\dagger A_\mu^2 A^\mu \phi$$

$$= [\partial_\mu \phi - ieA_\mu \phi]^\dagger [\partial^\mu \phi - ieA^\mu \phi]$$

$$= (D_\mu \phi)^\dagger (D^\mu \phi)$$

$$= \mathcal{L} \qquad (\checkmark)$$