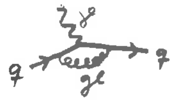
- three sources of NLO corrections

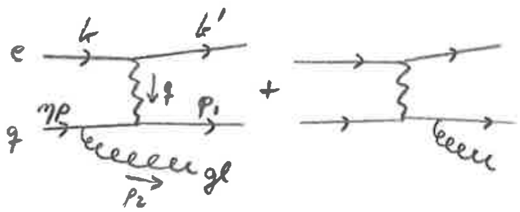
(1) $eq \to eq$ at 1-loop ("virtual corr.") 

(2) $eq \to eqg$ at tree-level 

(3) $eg \to eq\bar{q}$ at tree-level  ← completely new structure, not present in parton model

5.3.1 DIS @ NLO : $eq \to eqg$

- 2 diagrams, get $\langle|\mathcal{M}|^2\rangle$ by "crossing" from $e^+e^- \to q\bar{q}g$ (see §4.3.1)



$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4}\frac{1}{N_c}N_c \frac{8C_F e^4 Q_q^2 g_s^2}{(kk')(p_1 p_2)(\eta p \cdot p_2)}\left[(p_1\cdot k)^2 + (\eta p\cdot k)^2 + (p_1\cdot k')^2 + (\eta p\cdot k')^2\right]$$

→ phase space $\int d\Phi_3 = \underbrace{\int dQ^2 dx \frac{Q^2}{16\pi^2 s x^2}}_{\text{electron}} \int d\Phi_X$ here: X consists of 2 partons → non-triv.

$$\int d\Phi_X = \int_0^{2\pi} d\varphi \int_{-1}^{1} d(\cos\theta)\, \frac{1}{32\pi^2} = \int_0^{2\pi} d\varphi \int_0^1 dz\, \frac{1}{16\pi^2}$$

where (φ,θ) refer to direction of p_1 in CMS system of $\eta p + q$

alternatively, use Lorentz-invar. variable $z \equiv \frac{p_1\cdot p}{q\cdot p} = \frac{1}{2}(1-\cos\theta)$

→ partonic cross section

$$\frac{d^2\sigma(e+q)}{dQ^2\,dx} = \frac{1}{2\eta s}\frac{Q^2}{16\pi^2 s x^2}\int_0^{2\pi} d\varphi \int_0^1 dz \frac{1}{16\pi^2}\frac{2C_F e^4 Q_q^2 g_s^2}{(kk')(p_1 p_2)(\eta p\cdot p_2)}\left[\text{ see } \uparrow\right]$$

$$= \frac{Q^2 C_F \alpha^2 Q_q^2 \alpha_s}{2\eta x^2 s^2}\int_0^1 dz\, \frac{1}{2\pi}\int_0^{2\pi} d\varphi \frac{[\ldots]}{(kk')(p_1 p_2)(\eta p\cdot p_2)}$$

rewrite scalar products in terms of kinematic variables, perform φ-integration, use $\bar{x} \equiv x/\eta$

$$= \frac{Q^2 C_F \alpha^2 Q_q^2 \alpha_s}{2\eta x^2 s^2}\int_0^1 dz \frac{1}{y^2 Q^2}\left\{\left[1+(1-y)^2\right]\underbrace{\left(\frac{1+\bar{x}^2}{1-\bar{x}}\frac{1+z^2}{1-z} + 3 - z - \bar{x} + 11\bar{x}z\right)}_{\equiv f_2(\bar{x},z)} - y^2\left(8\bar{x}z\right)\right\}$$

will give a non-zero F_L

will give divergence in F_2 due to $\int dz$

• from cross section $\frac{d^2\sigma(e+p)}{dQ^2\,dx} \overset{(pg.\,62)}{=} \sum_q \int_0^1 d\eta\, f_q(\eta)\, \frac{d^2\sigma(e+q(\eta p))}{dQ^2\,dx}$

$\overset{(pg.\,61)}{=} \frac{2\pi\alpha^2}{xQ^4}\left\{\left[1+(1-y)^2\right]F_2(x,Q^2) - y^2 F_L(x,Q^2)\right\}$

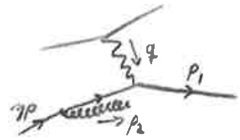
read off $F_2(x,Q^2) = \frac{C_F\alpha_s}{2\pi} \sum_q \int_0^1 d\eta\, f_q(\eta)\, Q_q^2 \underbrace{\boxed{\frac{xQ^4}{2\eta x^2 s^2 y^2}}}_{=\frac{x}{2\eta}} \int_0^1 dz\, \hat{f}_2\left(\frac{x}{\eta}, z\right)$

$\left(\bar{x} \equiv \frac{x}{\eta}\right)$ $= \frac{C_F\alpha_s}{2\pi} \sum_q \frac{1}{2}\int_x^\infty d\bar{x}\, f_q\left(\frac{x}{\bar{x}}\right)\frac{x}{\bar{x}}\, Q_q^2 \int_0^1 dz \left(\frac{1+\bar{x}^2}{1-\bar{x}}\,\frac{1+z^2}{1-z} + 3 - z - \bar{x} + 11\bar{x}z\right)$

$\frac{d\eta}{\eta} = -\frac{d\bar{x}}{\bar{x}}$

• consider the divergence in F_2

comes from $z \to 1$, where $z = \frac{p_1\cdot p}{q\cdot p}$



→ outgoing gluon collinear with incoming quark :

$\eta p\cdot p_2 = \eta p\cdot(\eta p + q - p_1) = \cancel{\eta^2 p^2} + \eta p\cdot q - \eta p\cdot p_1 = \eta p\cdot q\,(1-z)$

→ internal line becomes on-shell, causing the divergence:

$(\eta p - p_2)^2 = \cancel{\eta^2 p^2} - 2\eta p\cdot p_2 + \cancel{p_2^2}^{\,0} = -2\,\eta p\cdot q\,(1-z)$, quark propagator $\sim \frac{1}{1-z}$

Note also: coefficient of divergence $\sim \frac{1}{1-\bar{x}}$, diverges at $\bar{x}=1$,
when gluon is infinitely soft

• regulate divergence

consider transverse momentum $k_\perp$ of outgoing quark in CMS system of $(\eta p + q)$;
‖ it turns out that $k_\perp^2 = Q^2\left(\frac{\eta}{x}-1\right)z(1-z)$ ‖

$z\to 1$ means $k_\perp^2 \to 0$, so restricting $k_\perp^2 > \mu^2$ (with $\mu^2 \ll Q^2$)
regulates the divergence at $z \to 1$ ($\mu \to 0$ gives full result)

$\rightsquigarrow \int_0^1 dz \to \int_{z_-}^{z_+} dz$, where $z_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - 4\frac{\mu^2}{Q^2\left(\frac{\eta}{x}-1\right)}}\right)$; $\int_0^1 dz \approx \int_0^{1-\frac{\mu^2}{Q^2}} dz$

$\Rightarrow F_2(x,Q^2) = \frac{\alpha_s}{2\pi}\sum_q \frac{1}{2}\int_x^\infty d\bar{x}\, f_q\left(\frac{x}{\bar{x}}\right)\frac{x}{\bar{x}}\, Q_q^2 \left(2\,\hat{P}(\bar{x})\ln\frac{Q^2}{\mu^2} + \hat{R}(\bar{x})\right)$

↑ divergence ↑ finite; $\mu \to 0$ done

with "(unregularized) splitting function" $\hat{P}(x) \equiv C_F\frac{1+x^2}{1-x}$

describes probability distribution of outgoing quark in

note: have not (yet) removed the divergence,
only regulated it.