7. Outlook

→ have discussed aspects of a fascinating theory (QCD), with structures like non-Abelian gauge fields, coupling constant renormalization etc.

→ have seen in some examples that these abstract mathematical structures actually correspond to experimental observations

→ have so far mostly discussed perturbative QCD; how about non-perturbative phenomena / techniques ?

- can one actually <u>solve QCD</u> analytically ?
  "holy grail" for field theorists !
  prize money : \$1 M ; see www.claymath.org/millennium
  as a first step: try to solve pure YM (symm's could help!)
  ((or, even more symmetric, supersymmetric YM (SYM)))

  (one) goal : take "idealized" QCD, u+d+gl, all massless
  $$\mathcal{L} = \bar{u}\, i\not{D} u + \bar{d}\, i\not{D} d - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$
  calculate e.g. $\frac{m_{\text{g-Meson}}}{m_{\text{Proton}}} \sim \{ \frac{1}{2}, \pi, \ln 2, \zeta(3), \dots \}$

  (($\mathcal{L}$ has dimless parameter g^2; but $g^2(\mu)$ due to renormalization; dimensionful scale Λ_{QCD} where $\frac{g^2(\Lambda_{QCD})}{4\pi} \sim 1$; so each $m \sim \Lambda_{QCD}$))

- note that <u>perturbation theory</u> is somewhat <u>unnatural</u> for solving a highly symmetric gauge theory such as YM :
  split $F^a_{\mu\nu} F^{a\mu\nu} \to (\partial A - \partial A)^2 + A^3 + A^4$
  $= \text{harmonic osc.} + \text{rest} = \mathcal{L}_0 + \mathcal{L}_{\text{interaction}}$

  gauge invar. — not g.i. — not g.i.

  ⇒ clearly not optimal; solvable/integrable systems <u>need</u> symmetry !

- can one work with QCD in the regime where the strong coupling is actually strong ?

  → big open question: confinement

  would like to derive the complete force between quarks from $\mathcal{L}_{QCD}$

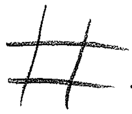
  weak coupling limit: Coulomb potential w/ running coupling ✓✓

  strong coupling limit: linear potential, confines color

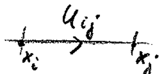
  (("string" ?)) derivation needs new tools:

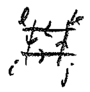
- [Wilson, Phys. Rev. D 10 (1974) 2445]: lattice gauge theory

  violate Lorentz-invariance, not gauge invariance

  formulate theory on 4d Euclidean spacetime lattice  $\}a$,

  perform continuum limit $a \to 0$ in the end, to recover 4d rotational invariance and (after Wick rot.) Lorentz invar.

  fundamental variables: line elements  ; U_{ij} = unitary, $N\times N$ matrix

  gauge invariant quantity: plaquette  $\operatorname{tr}(U_{ij}U_{jk}U_{kl}U_{li})$

  invariant under local trafo: $U_{ij} \to V_i^+ U_{ij} V_j$

  def $Z = \int \mathcal{D}U \; e^{\frac{1}{2g^2}\sum_{\text{plaq}} \operatorname{Re}\operatorname{tr}(UUUU)}$ (Wilson)

  → is this equivalent to YM ? yes, after $a \to 0$:

  def $U_{ij} = V_i^+ e^{ia A_\mu(x)} V_j$, where $x \equiv \frac{x_i + x_j}{2}$, $\mu \equiv i \to j$ direction

  ((along $\hat{\mu} = \frac{x_j - x_i}{a}$))

  then $UUUU = e^{ia^2 F_{\mu\nu} + O(a^3)}$

  and $\operatorname{Re}\operatorname{tr}(UUUU) \overset{!}{=} \operatorname{Re}\operatorname{tr}\left\{1 + ia^2 F_{\mu\nu} - \frac{1}{2}a^4 F_{\mu\nu}F_{\mu\nu} + \dots\right\}$

  $= \operatorname{tr} 1 - \frac{a^4}{2}\operatorname{tr} F_{\mu\nu}F_{\mu\nu} + \dots$

  ⇝ beautiful formulation: no gauge fixing, no ghosts

  ⇝ as a challenge, try to incorporate fermions!

  highly nontrivial problem, ongoing research, ...

- for practical purposes, lattice gauge theory allows for numerical computations.

  → lattice Monte Carlo methods;
  huge world-wide efforts, development of algorithms and computers.

  → principal theoretical tool for quantitative calculations in hadron physics.
  get e.g. mass spectrum of low-lying mesons + baryons to ~10%
  ((as we have seen in §1.2))

- marriage of relativity + QM ⇒ QFT
  " + statistical physics ⇒ thermal QFT

  → hot QCD is very interesting (phase transitions, ...),
  relevant (early universe, ...),
  conceptually clear (hadrons melt → quark-gluon plasma)

  ⇒ can be treated analytically (weak-coupling)
  numerically (lattice Monte Carlo)
  experimentally (heavy-ion collisions)

- as in other QFT's, QCD allows for interesting non-perturbative objects (exact sol's of eom; solitons, vortices, monopoles, instantons, ...) and non-perturbative methods (large-N expansion, ...)

- what is next?

  → master's thesis, ask everyone, get valuable insight into Bielefeld research

  → lectures in WS 11: lattice gauge theory (Karsch)
  supersymmetry (Kögerler)
  electroweak physics (Brauner)