2.3 QCD and its symmetries

- Quantum Chromodynamics (QCD) is a Yang-Mills theory with gauge group $SU(3)$.
  - matter fields (the ψ above) are quarks; they are in the fundamental representation of $SU(3)$, have spin $\frac{1}{2}$; there are six types ("flavors") of quarks: u, d, c, s, t, b; index of gauge group is called color index
    $\Rightarrow$ write as $\psi^{\alpha,A}$; color index $\alpha = 1,2,3$; flavor index $A = u,d,c,s,t,b$
  - the $3^2-1=8$ vector fields (or gauge bosons) A_μ^a, $a=1,\dots,8$ are called gluons

- $$\mathcal{L}_{QCD} = \bar{\psi}^{\alpha,A}\Big(i\gamma^\mu\big(\partial_\mu\delta_{\alpha\beta} - ig A_\mu^a T^a_{\alpha\beta}\big) - m_A\delta_{\alpha\beta}\Big)\psi^{\beta,A} - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu}$$

  ((sum over color indices α,β; sum over flavor index A))

  $T^a_{\alpha\beta}$: generators of $SU(3)$ in fundamental rep.

  m_A: each quark flavor can have a different mass

- sometimes, it is useful to consider the generalizations
  $SU(3) \to SU(N_c)$ $\Rightarrow$ colors: $\alpha,\beta = 1,\dots,N_c$; gluons: $a = 1,\dots,N_c^2-1$
  6 quark flavors $\to N_f$ quark flavors $\Rightarrow A = 1,\dots,N_f$

- QCD possesses not only the exact local $SU(N_c)$ color symmetry, but has also important approximate global symmetries:
  $\rightarrow$ consider (x-independent) rotations in flavor space
  ((note: global phase-redefinition for each flavor ($A=u,d,\dots$) separately $\Rightarrow \mathcal{L}_{QCD}$ invariant $\Rightarrow$ quark number conserved))
  $\rightarrow$ rotations between different flavors make sense if some masses are (approximately) degenerate
  ((note: in nature, $m_u \sim 3$ MeV, $m_d \sim 5$ MeV
  $\Rightarrow m_d - m_u \ll m_{Hadron} \sim 150$ MeV $\Rightarrow \mathcal{L}$ has increased symmetry))

$\rightarrow$ assume e.g. $\underline{m_u \approx m_d}$ $\Rightarrow$ $M \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \approx m_u \mathbb{1}_{2\times2}$

write $q \equiv \begin{pmatrix} q^u \\ q^d \end{pmatrix}$, then $\mathcal{L}_{QCD} \ni \bar{q}(i\not{D} - M)q$

is invariant under $q \rightarrow \underbrace{e^{i\sum_{j=0}^{3}\alpha_j \sigma^j}}_{\in\, U(2) \,=\, U(1) \otimes SU(2)} q$ $\quad$ (($\sigma^{1,2,3}$ = Pauli, $\sigma^0 \equiv \mathbb{1}_{2\times2}$))

quark number symmetry (see above) ↑ $\quad$ ↑ "isospin symmetry" exact only if $m_u = m_d$

((note: these symmetries are, via Noether's theorem, associated with vector currents $J^i_\mu = \bar{q}\gamma_\mu \sigma^i q$, hence often $SU(2)_V$))

((note: if e.g. $m_u \approx m_d \approx m_s$ is a useful approximation, then symmetry is enhanced, $SU(3)_V$; etc))

$\rightarrow$ for massless flavors, the symmetry becomes even larger: $M \equiv \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

use left- and right-handed projectors

$$\gamma_{L/R} \equiv \frac{1 \mp \gamma^5}{2} \qquad \left(\left(\Rightarrow \gamma_L^2 = \gamma_L,\ \gamma_R^2 = \gamma_R,\ \gamma_L\gamma_R = 0 \right)\right)$$

decompose $q^u = (\gamma_L + \gamma_R)q^u \equiv q^u_L + q^u_R$ etc

now $q_L \equiv \begin{pmatrix} q^u_L \\ q^d_L \end{pmatrix}$, and $\mathcal{L}_{QCD} \ni \bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R$

((note: $M \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ would have coupled L, R: $\mathcal{L} \ni -q_L^\dagger M q_R$ + h.c.))

$\Rightarrow$ independent transformations $q_L \rightarrow U_L q_L$, $q_R \rightarrow U_R q_R$ permitted!

$\rightsquigarrow$ $U(2)_L \otimes U(2)_R$ symmetry

$= U(1)_L \otimes U(1)_R \otimes SU(2)_L \otimes SU(2)_R$, called chiral symmetry

((since acting separately on L, R))

((note: the symmetry $SU(N_f)_L \otimes SU(N_f)_R$ is sometimes rewritten as the product $SU(N_f)_V \otimes SU(N_f)_A$ "axial",

using $Q \equiv \begin{pmatrix} q_L \\ q_R \end{pmatrix}$, $\mathcal{L}_{QCD} \ni \bar{Q} i\not{D} Q$,

invariant under $Q \rightarrow e^{i\alpha^a T^a} Q$ and $Q \rightarrow e^{i\beta^a T^a \gamma^5} Q$))

T^a: generators of $SU(N_f)$ flavor symmetry in fundamental rep. $\quad$ to check this invariance, see (left)

$\{\gamma^5, \gamma^\mu\} = 0$, $(\gamma^5)^\dagger = \gamma^5$

$\Rightarrow \gamma^\mu e^{i\beta\gamma^5} = e^{-i\beta\gamma^5}\gamma^\mu$ via $e^a = \sum \frac{a^n}{n!}$

$\Rightarrow \gamma^0\gamma^\mu e^{i\beta\gamma^5} = e^{i\beta\gamma^5}\gamma^0\gamma^\mu$

$\Rightarrow \bar{\psi}\not{\partial}\psi = \psi^\dagger\gamma^0\gamma^\mu\partial_\mu\psi \rightarrow \psi^\dagger e^{-i\beta\gamma^5}\gamma^0\gamma^\mu\partial_\mu e^{i\beta\gamma^5}\psi = \bar{\psi}\not{\partial}\psi$

- similar to the above approximate global symmetries for light quarks (neglecting effects of order m_q), can also consider <u>heavy quark symmetries</u> (neglecting effects of order $1/m_Q$).
  - → systematics, "heavy quark effective theories", see e.g. [M. Neubert, Phys. Rept. 245 (1994) 259]

- other important exact symmetries of QCD are the <u>discrete global symmetries</u>: C, P, T
  ((these agree with the <u>observed properties</u> of the strong interactions; for tests and limits, see Particle Data Group, pdg.lbl.gov))
  - → analysis of $\mathcal{L}_{QCD}$ under C, P, T is complicated (at quantum level) due to the possible dim-4 operator we had discovered (see pg. 14)

$$\mathcal{L}_\theta = \underbrace{\frac{\theta g^2}{32\pi^2}}_{} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \quad , \text{ where } \quad \tilde{F}^{a\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma}$$

conventional normalization ; on pg. 14: "c"

  - → $\mathcal{L}_\theta$ would violate both P and T, in contradiction to observations
    ⇒ set $\theta = 0$, or at least $\theta \ll 1$?!
    ($\ll 1$: could be regenerated by known CP effects in weak int.)
  - → actually, $F^a_{\mu\nu} \tilde{F}^{a\mu\nu} = \partial_\alpha \left\{ 2\varepsilon^{\alpha\beta\gamma\delta} A^a_\beta \left(\partial_\gamma A^a_\delta - \frac{2}{3} g f^{abc} A^b_\gamma A^c_\delta \right) \right\}$
    is a total derivative
    contributes only surface term to action $S = i \int d^4x\, \mathcal{L}$
    therefore plays no role in perturbative QCD
  - → however, $\mathcal{L}_\theta$ can have real physical effects due to non-perturbative effects (QCD vacuum can have non-trivial topology ⇒ surface terms contribute; the $\{\ldots\}$ is <u>not</u> gauge-invariant)
    [see e.g. Erice lectures by S. Coleman (1977), F. Wilczek (1983)]
  - → problem: observations tell $\theta < 10^{-9}$ (neutron dipole moment)
    "naturally", θ should be large (coming from <u>strong</u> interactions)
    ⇒ "<u>strong CP problem</u>"
    ⇒ several proposed solutions; e.g. Peccei-Quinn-symmetry
    ⇒ new particles: <u>Axions</u>