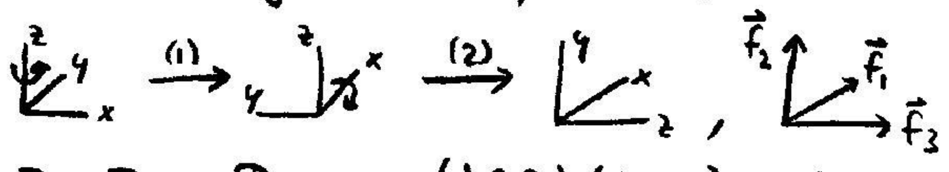
Klausur zu EMTP I (12. Feb. 2007) Lösung

(1) $v = \frac{ds}{dt} = \frac{n \cdot d\varphi \cdot R}{dt} = \frac{n \cdot \frac{2\pi}{17} \cdot \frac{0.8\,m}{2}}{1/_{50}\ s} = n \cdot \frac{40\pi}{17}\,\frac{m}{s} \quad \left(\left(\approx n \cdot 7.39\,\frac{m}{s}\right)\right)$, $n = 0, 1, 2, \dots$

(2) (a) $E_{z=h} = E_{z=\ell}$, $mgh + \frac{k}{2}(\ell-h)^2 = \frac{m}{2}v^2 + mg\ell \Rightarrow v = \sqrt{\ell-h}\sqrt{\frac{k}{m}(\ell-h)-2g}$

$K_{min} = 2gm/(\ell-h)$, damit $\sqrt{\geq 0}$

(b) $m\ddot{z} = -mg + k(\ell - z)$, $\dot{z}(0) = 0$, $z(0) = h$

(3) 

$D = D_{x,\frac{\pi}{2}} D_{z,\frac{\pi}{2}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, $D\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(4) (a) $\dot{f} = -\lambda(1+\omega t)$, $f = const_t - \lambda(t + \frac{1}{2}\omega t^2)$, $f(0) = const_t = \ln(v_0) \Rightarrow v(t) = v_0 e^{-\lambda(t+\frac{1}{2}\omega t^2)}$

(b) $\dot{f} = -\lambda/(1+\omega t)$, $f = const_t - \frac{\lambda}{\omega}\ln(1+\omega t)$, $f(0) = const_t = \ln(v_0) \Rightarrow v(t) = v_0\,(1+\omega t)^{-\lambda/\omega}$

(5) (a) $v\dot{v} = (\frac{1}{2}v^2)^\bullet = -\lambda = (const_t - \lambda t)^\bullet \Rightarrow v(t) = \sqrt{2(const_t - \lambda t)} = \sqrt{v_0^2 - 2\lambda t}$

(b) $v^n\dot{v} = \left(\frac{1}{n+1}v^{n+1}\right)^\bullet = -\lambda = (const_t - \lambda t)^\bullet \Rightarrow v(t) = \left[(n+1)(const_t - \lambda t)\right]^{\frac{1}{n+1}} = \left[v_0^{\,n+1} - (n+1)\lambda t\right]^{\frac{1}{n+1}}$

(6) $\vec{K} = -(\partial_x V, \partial_y V, \partial_z V)$, $K_1 = -\alpha x^2 \stackrel{!}{=} -\partial_x V = -\partial_x\left(\frac{\alpha}{3}x^3 + f(y,z)\right)$, $K_2 = \beta z \stackrel{!}{=} -\partial_y V = -\partial_y\left(\frac{\alpha}{3}x^3 - \beta yz + f(z)\right)$,

$K_3 = \gamma y \stackrel{!}{=} -\partial_z V = -\partial_z\left(\frac{\alpha}{3}x^3 - \beta yz + f(z)\right) = \beta y + f'(z) \Rightarrow \beta \stackrel{!}{=} \gamma$, $V = \frac{\alpha}{3}x^3 - \beta yz$

(7) (a) $0 \stackrel{!}{=} (1-\lambda)^2 - 4 = \lambda^2 - 2\lambda - 3 = (\lambda-3)(\lambda+1)$, $\lambda_1 = 3$, $\lambda_2 = -1$.

$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} r \\ s \end{pmatrix} = 2\begin{pmatrix} s-r \\ r-s \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \vec{f}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, ... $\vec{f}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\vec{f}_1 \cdot \vec{f}_2 = 0$ ✓, $D = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

(b) $0 \stackrel{!}{=} (3-\lambda)(4-\lambda)(5-\lambda) - (3-\lambda)4 - (5-\lambda)4 = -\lambda^3 + 12\lambda^2 - 39\lambda + 28$, $\lambda_1 = 1$ (raten)

$= (1-\lambda)(28 - 11\lambda + \lambda^2)$, $\lambda_{2/3} = \frac{11}{2} \pm \sqrt{\frac{121}{4} - 28} \Rightarrow \lambda_2 = 4$, $\lambda_3 = 7$, $3+4+5 = 1+4+7$ ✓

(8) (a) $\ln(\sqrt{4+\varepsilon^2}) = \ln(2\sqrt{1+\varepsilon^2/4}) = \ln 2 + \frac{1}{2}\ln(1+\varepsilon^2/4) = \ln 2 + \frac{\varepsilon^2}{8} + O(\varepsilon^4)$

(b) $\ln\left(2\varepsilon + \frac{(2\varepsilon)^2}{2!} + \frac{(2\varepsilon)^3}{3!} + \dots\right) = \ln(2\varepsilon) + \ln\left(1 + [\varepsilon + \frac{2}{3}\varepsilon^2 + \dots]\right) = \ln(2\varepsilon) + [\;] - \frac{1}{2}[\;]^2 + \dots$

$= \ln(2\varepsilon) + \varepsilon + \frac{1}{6}\varepsilon^2 + O(\varepsilon^3)$

(c) $= (b)$, oder: $\varepsilon + \ln(2\sinh(\varepsilon)) = \varepsilon + \ln\left(2\varepsilon + 2\frac{\varepsilon^3}{3!} + \dots\right) = \varepsilon + \ln(2\varepsilon) + \frac{1}{6}\varepsilon^2 + O(\varepsilon^3)$

(9) $f = \ln(x)$, $f' = \frac{1}{x}$, $f_u = e^x$; $f_u' = \partial_x e^x = \frac{1}{f'(f_u(x))} = \frac{1}{1/e^x} = e^x$

(10) $ER^{(0)}$: $\ddot{x}^{(0)} = 0$, $\dot{x}^{(0)}(0) = 0$, $x^{(0)}(0) = a \Rightarrow x^{(0)} = a$

$ER^{(1)}$: $\ddot{x}^{(1)} = -\omega^2 a$, $\dot{x}^{(1)}(0) = 0$, $x^{(1)}(0) = 0 \Rightarrow x^{(1)} = -\frac{1}{2}\omega^2 a t^2$

$ER^{(2)}$: $\ddot{x}^{(2)} = -\omega^2 x^{(1)} = +\omega^4 \frac{a}{2} t^2$, $\dot{x}^{(2)}(0) = 0$, $x^{(2)}(0) = 0 \Rightarrow x^{(2)} = \omega^4 \frac{a}{2}\frac{t^4}{3 \cdot 4}$

$\Rightarrow x(t) \approx a\left[1 - \frac{1}{2}(\omega t)^2 + \frac{1}{4!}(\omega t)^4 + \dots\right] \stackrel{\wedge}{=} a \cdot \cos(\omega t)$ ✓

Ergebnisse: ab 16. Feb.; online + Aushang (E6).