(39) (a) $\boxed{\dot v = -\alpha v + \beta \frac{1}{\varepsilon} \ln\left(\frac{v}{v_0}\right), \quad v(0) = v_0}$ ●

(b) Ok, weil $v < v_0$ wird, d.h. ln negativ ◐

(c) $[\beta] = [\dot v t] = [v] \Rightarrow \beta \ll v_0$ ◐

(d) $\boxed{\dot v^{(0)} = -\alpha v^{(0)}, \quad v^{(0)}(0) = v_0} \Rightarrow v^{(0)}(t) = v_0 e^{-\alpha t}$ ●

$\dot v^{(1)} = -\alpha v^{(1)} + \beta \frac{1}{\varepsilon} \ln\left(\frac{v^{(0)}}{v_0}\right)$

$\boxed{\dot v^{(1)} = -\alpha v^{(1)} - \alpha\beta, \quad v^{(1)}(0) = 0}$ ◐ $\Rightarrow v^{(1)}(t) = -\beta + \beta e^{-\alpha t}$ ◐

(40) (a) $\partial_x \arctan(x) = \frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-x^2)^n = 1 - x^2 + x^4 - \dots$ ◐

$\Rightarrow \arctan(x) = \text{const.} + x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \dots, \quad \text{const} = \arctan(0) = 0$ ◐

(b) $\tan(x) = \dfrac{x - x^3/3! + x^5/5! - \dots}{1 - x^2/2! + x^4/4! - \dots} = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)\left(1 + \frac{x^2}{2!} - \frac{x^4}{4!} + \left(\frac{x^2}{2!}\right)^2 + \dots\right)$

$= x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots$ ●

(c) $mc^2(1-\varepsilon^2)^{-1/2} = mc^2\left(1 - \frac{1}{2}(-\varepsilon^2) + \frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(-\varepsilon^2)^2 + \dots\right) = mc^2 + \frac{1}{2}mv^2 + \frac{3}{8}\frac{mv^4}{c^2} + \dots$ ◐

(d) $mc^2\sqrt{1 + \frac{p^2}{m^2c^2}} = mc^2(1+\varepsilon^2)^{1/2} = mc^2\left(1 + \frac{1}{2}\varepsilon^2 + \frac{1}{2}\frac{1}{2}(-\frac{1}{2})\varepsilon^4 + \dots\right) = mc^2 + \frac{1}{2}\frac{p^2}{m} - \frac{1}{8}\frac{p^4}{m^3c^2}$ ◐

(41) (a) $\left.\begin{array}{l} M\dot v_1 = -M\alpha(v_1 - v_2) \\ M\dot v_2 = -M\alpha(v_2 - v_1) \end{array}\right\} \quad \dot{\vec v} = -\alpha H \vec v, \quad H = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ ◐

(b) EV: $f_{1/2} = \frac{1}{\sqrt 2}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}$ ◐ ((und EW: $\lambda_{1/2} = \begin{cases} 0 \\ 2 \end{cases}$))

Ansatz $\vec v = A(t)\vec f_1 + B(t)\vec f_2$ in ER:

$\dot A \vec f_1 + \dot B \vec f_2 = -\alpha B\, 2 \vec f_2, \quad A(0)\vec f_1 + B(0)\vec f_2 = \frac{v_0}{\sqrt 2}(\vec f_1 + \vec f_2)$

$\Rightarrow \boxed{\dot A = 0, \quad A(0) = \frac{v_0}{\sqrt 2}; \quad \dot B = -2\alpha B, \quad B(0) = \frac{v_0}{\sqrt 2}}$ ◐

Lsg $\Rightarrow A(t) = \frac{v_0}{\sqrt 2}; \quad B(t) = \frac{v_0}{\sqrt 2} e^{-2\alpha t}$ ◐

also $\vec v(t) = \frac{v_0}{\sqrt 2}\frac{1}{\sqrt 2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{v_0}{\sqrt 2}\frac{1}{\sqrt 2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{v_0}{2}\begin{pmatrix} 1 + e^{-2\alpha t} \\ 1 - e^{-2\alpha t} \end{pmatrix} = v_0 e^{-\alpha t}\begin{pmatrix} \cosh(\alpha t) \\ \sinh(\alpha t) \end{pmatrix}$ ◐

und $\vec v(t \to \infty) = \frac{v_0}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ◐, sehr beruhigend!

(c) $\vec v(t) = e^{-\alpha H t} v_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ◐

$= e^{-\alpha t}\, \boxed{e^{+\alpha R t}}\, v_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \text{mit } R = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad R \cdot R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbb{1}$ ◐

$e^{+\alpha R t} = \mathbb{1} + \alpha t R + \frac{1}{2!}(\alpha t)^2 \underbrace{R \cdot R}_{=\mathbb{1}} + \frac{1}{3!}(\alpha t)^3 R \underbrace{R \cdot R}_{=\mathbb{1}} + \frac{1}{4!}(\alpha t)^4 \underbrace{R \cdot R \cdot R \cdot R}_{=\mathbb{1}\cdot\mathbb{1} = \mathbb{1}} + \dots$

$= \mathbb{1}\left(1 + \frac{(\alpha t)^2}{2!} + \frac{(\alpha t)^4}{4!} + \dots\right) + R\left(\alpha t + \frac{(\alpha t)^3}{3!} + \dots\right)$

$= \mathbb{1} \cosh(\alpha t) + R \sinh(\alpha t)$ ◐

$= v_0 e^{-\alpha t}\left[\begin{pmatrix} \cosh(\alpha t) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \sinh(\alpha t) \end{pmatrix}\right]$ ◐ wie (b) ✓