Introduction to Astronomy
Exercises week 14

24 January 2020

1. In order to get a feeling for Olber's paradox, consider you're in a forest and try to evaluate how much of your view is blocked by trees. To do so, consider what fraction of directions (i.e. of the 360 degrees you can look around yourself) are blocked by trees, as a function of distance. (Clearly nearby you'll only see few trees and the fractional obstruction of your view is limited; considering longer distances, this blockage will accumulate through the combined effect of near and far trees.) Consider the number density of trees (measured in number of trees per square metre) to be N and the diameter of these trees to be D (typically). Express the fraction of directions blocked as $s(r)$ where r is the distance considered. Use your result to calculate the percentage of the horizon that is covered if there is on average 1 tree per square metre, with a typical diameter of 20 cm and you're in the middle of a forest with a radius of 20 m.

2. Hubble's law can be used (in a somewhat simplistic way) to calculate distances based on radial velocities. Interpreting the redshift as $z = v_{\rm rad}/c$, calculate the redshift, distance, diameter and absolute magnitude of a galaxy that has a radial velocity of 6940 km/s, an apparent magnitude of 14.4 and an angular diameter of 1.3 arcminutes.

3. In the radiation-dominated early Universe (which we can approximate with a flat geometry), the temperature of the Universe evolved with time as follows:

$$T(t) = \left(\frac{3c^2}{32\pi G a}\right)^{1/4} t^{-1/2} = t^{-1/2} \times 1.52 \times 10^{10}\,\mathrm{K s^{1/2}},$$

with G Newton's gravitational constant and a is the radiation constant ($a = 4\sigma/c$). Also, the Robertson-Walker scale factor R evolved as

$$R(t) = \left(\frac{32\pi G a T_0^4}{3c^2}\right)^{1/4} t^{1/2} = t^{1/2} \times 1.80 \times 10^{-10}\,\mathrm{s^{-1/2}},$$

with T_0 the temperature of the CMB. If the scale factor relates to redshift accoding to $1 + z = R(t)^{-1}$, then calculate the redshift at which He nuclei were formed (this happens at a temperature of 10^9 K).