Introduction to Astronomy
Exercises week 12

10 January 2020

1. Assuming stars are uniformly distributed in space and there is no interstellar extinction, show that the number of stars brighter than apparent magnitude m, is:

$$N(m) = N_0 \times 10^{0.6m},$$

with N_0 a proportionality constant. (Hint: start off by assuming all stars have identical absolute magnitude M, then calculate the distance for stars of apparent magnitude m. Go from there.)

> **Solution:**
> Assuming all stars have absolute magnitude M and given the definition of absolute magnitude, the distance of a star with apparent magnitude m would be:
>
> $$D = 10\,\mathrm{pc}\,10^{(m-M)/5}.$$
>
> All stars that are within a sphere of radius D, would then be brighter than m. This means:
>
> $$N(m) = \rho\frac{4}{3}\pi D^3 = \frac{4\pi}{3}\rho\,(10\,\mathrm{pc})^3\,\frac{10^{0.6\,\mathrm{m}}}{10^{0.6\,\mathrm{M}}},$$
>
> with ρ the stellar density. Since $10^{-0.6M}$ is a constant, we end up with:
>
> $$N(m) \propto 10^{0.6m},$$
>
> or $N(m) = N_0 \times 10^{0.6m}$ as expected. Note that this result does not depend on M and therefore should hold even when M is not constant, but variable.

2. The "3-kpc arm" is a feature in our Galaxy that appears to be expanding away from the Galactic Centre with $V_{\rm exp} = 50\,\mathrm{km/s}$. Assume that this feature is a donut-shaped ring with a radius of 3 kpc and a total mass of $6 \times 10^7\,\mathrm{M_\odot}$. Find the kinetic energy of this feature. If this energy is supplied by supernovae (SNe), each with an available energy of 10^{44} J, how many SNe would be needed to supply the kinetic energy?

> **Solution:**
> The kinetic energy is easily obtained to be:
>
> $$E_{\rm kin} = \frac{MV^2}{2} = \frac{6\times10^7 \times 1.989\times10^{30}\,\mathrm{kg}\left(5\times10^4\,\mathrm{km/s}\right)^2}{2} = 1.5\times10^{47}\,\mathrm{J}.$$
>
> This would be equivalent to 1500 supernovae.

3. We observe an HI cloud at Galactic longitude $l = 111°$. The cloud has a radial velocity of $v_{\rm R} = -40\,\mathrm{km/s}$. Calculate the distance to the cloud, assuming a constant Galactic rotation velocity of 240 km/s and taking $R_0 = 8\,\mathrm{kpc}$ for the distance between the Solar System and the Galactic Centre.

Solution:

Based on the picture below, this question can be reduced to geometry. Specifically, the radial velocity can be calculated as:

$$V_{\rm r} = V \sin\alpha - V_0 \sin l$$

where V is the velocity of the cloud in its rotation around the Galactic centre (given as $240\,\mathrm{km/s}$ on the formula sheet), α is the angle between the line-of-sight from the Earth to the cloud and the line connecting the cloud and the Galactic centre, l is the Galactic longitude of the cloud and V_0 is the rotational velocity of the Solar System around the Galactic centre (which is given to be constant and equal to $V = 240\,\mathrm{km/s}$). So this equation can give us α:

$$\alpha = \arcsin\left[\frac{1}{V}\left(V_{\rm r} + V_0 \sin l\right)\right] = 50^\circ$$

This leads us to determine β:

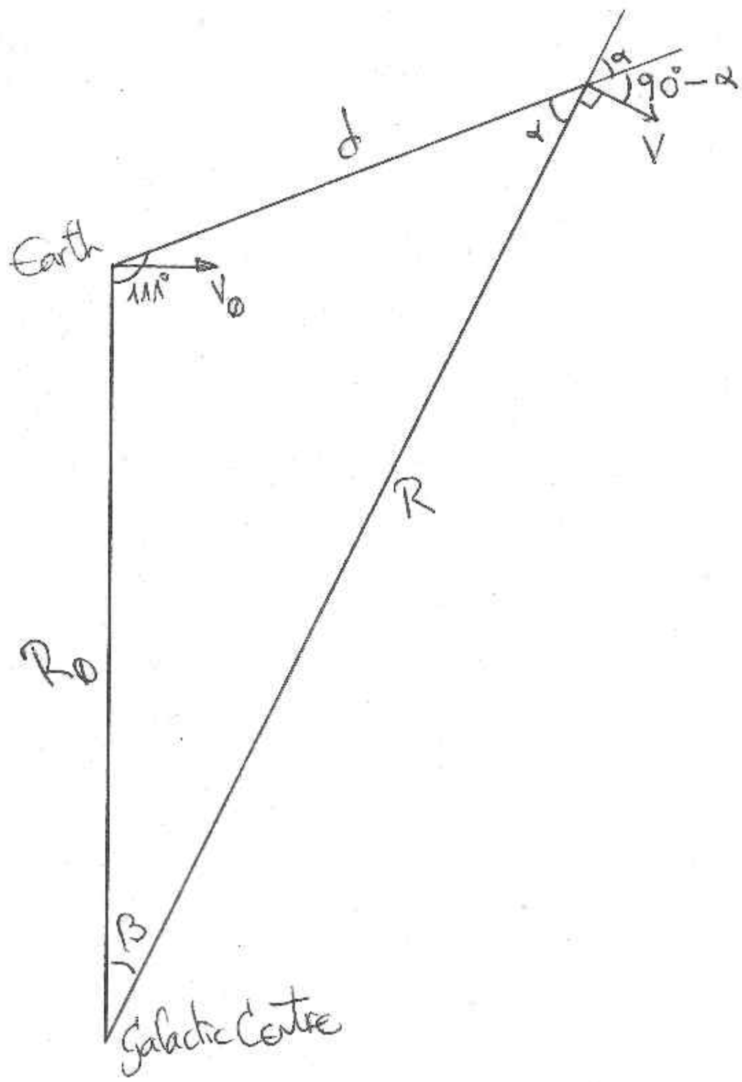
$$\beta = 180^\circ - l - \alpha = 180^\circ - 111^\circ - 50^\circ = 19^\circ$$

Next, using the law of sines, we can determine the distance d:

$$\frac{\sin\beta}{d} = \frac{\sin\alpha}{R_0}$$

hence:

$$d = \frac{\sin\beta}{\sin\alpha} R_0 = \frac{\sin 19^\circ}{\sin 50^\circ} 8\mathrm{kpc} = 3.4\mathrm{kpc}.$$



4. For stars on the Solar circle (i.e. with a distance to the Galactic Centre of 8 kpc, like our Sun), the proper motion caused by Galactic rotation is independent from the stellar distance. Prove this by determining the value of this proper motion, assuming the constants given in the previous question.

Solution:

If we consider the velocity perpendicular to the line connecting the Earth with the star, we get the proper motion:

$$\mu d = V \sin(90 - \alpha) - V_0 \sin(l - 90).$$

Now, since the star is also a distance R_0 away from the Galactic Centre, we know that $\alpha = l$. Furthermore, it is given that $V = V_0$. Using this and the facts that $\sin(90 - \alpha) = \cos(\alpha)$ and $\sin(l - 90) = -\cos(l)$, we get:

$$\mu d = V_0 \cos(\alpha) + V_0 \cos(l) = 2V_0 \cos(\alpha).$$

Using the sine rule again, and using $\beta = 180 - 2\alpha$, we have:

$$d = \frac{R_0}{\sin(\alpha)} \sin(\beta) = R_0 \frac{\sin(2\alpha)}{\sin(\alpha)}.$$

Since $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$, we have: $d = 2\cos(\alpha) R_0$. Using this in the equation for μ, we get:

$$\mu = \frac{2V_0 \cos(\alpha)}{2\cos(\alpha) R_0} = \frac{V_0}{R_0}.$$

With the values given, this equates to:

$$\mu = \frac{240\,\text{km/s}}{8\,\text{kpc}} = 30 \frac{\text{km/s}}{\text{kpc}} = 9.72 \times 10^{-16}\,\text{rad/s} \approx 0.006''\,\text{yr}^{-1}.$$