Introduction to Astronomy
Exercises week 10

13 December 2019

1. A globular cluster contains a million stars, each with an absolute magnitude equal to that of the Sun. If the cluster is 10 kpc away, calculate the total apparent magnitude of the cluster.

2. The Pleiades cluster contains 230 stars within a radius of 4 pc. Assume that all stars weigh one Solar mass and that they are all separated by 2 pc.
   (a) Calculate the average potential and kinetic energy for each star and use the virial theorem ($\langle E_{\rm kin}\rangle = -0.5\langle E_{\rm pot}\rangle$) to estimate the typical velocity of stars in this cluster.
   (b) Now calculate the escape velocity of this cluster, at a distance of 2 pc.

3. One of the (distance-insensitive) ways to determine the age of a globular cluster, is by measuring the offset between its turnoff point and its horizontal branch: $\Delta V = M_{\rm V}({\rm TO}) - M_{\rm V}({\rm HB})$. However, this measure is not only dependent on the age, but also on the metallicity of the cluster:

$$\Delta V = 2.7 \log\left(\frac{t}{\rm Gyr}\right) + 0.13[{\rm Fe/H}] + 0.59.$$

   Estimate by how much the metallicity [Fe/H] must be off in order to produce a 10% error in the age of the cluster. Note that the metallicity is defined as follows:

$$\left[\frac{\rm Fe}{\rm H}\right] = \log\left(\frac{N_{\rm Fe}}{N_{\rm H}}\right)^{\rm Star} - \log\left(\frac{N_{\rm Fe}}{N_{\rm H}}\right)^{\rm Sun}$$

   where log is the logarithm base 10.