Introduction to Astronomy
Exercises week 9

6 December 2019

1. The Crab pulsar (PSR B0531+21) has a spectral index of -3 over the frequency range from 10 MHz to 10 GHz (i.e. $S \propto \nu^{-3}$). If the measured flux at 400 MHz is 650 mJy and the spin-down luminosity $\mathrm{d}E/\mathrm{d}t$ is given by

$$\mathrm{d}E/\mathrm{d}t = 4\pi^2 I P^{-3} \mathrm{d}P/\mathrm{d}t$$

with moment of inertia $I = 10^{45}\,\mathrm{g\,cm^2}$, spin period $P = 33\,\mathrm{ms}$ and spindown $\mathrm{d}P/\mathrm{d}t = 4.23 \times 10^{-13}\mathrm{s/s}$, calculate what fraction of energy loss the radio emission (between 10 MHz and 10 GHz) accounts for. The Crab pulsar is at a distance of approximately 2 kpc. (Hint: calculate the energy loss derived from the spin-down luminosity. Also calculate the total power emitted in the radio by integrating the flux over frequency and over a sphere, assuming isotropic emission. Then compare these two values.)

2. A Cepheid variable has a period of 20 days and a mean apparent magnitude of 20 mag. What is its distance?

3. Cassiopeia A is a supernova remnant (SNR) with an angular diameter of $5.5'$ at a distance of 3 kpc.
   (a) If the observed expansion velocity is 6.8×10^6 m/s, calculate the expected age of the remnant, assuming a constant expansion velocity.
   (b) If we have a resolution of 50 mas[1], how long would we need to observe in order to detect the expansion of the nebula?

4. A variable star changes its magnitude by 2 mag. If its effective temperature is 6000 K at the maximum and 5000 K at the minimum, how much does its radius change?

[1] mas = milli-arcsecond