Introduction to Astronomy
Exercises week 6

15 November 2019

1. Flux density can be defined in two different ways: either it can be considered to be the power per unit collecting area and unit bandwidth, or as the power per unit collecting area and unit wavelength. Both definitions are common, depending on which area of astronomy you consider, so it is important to know how to convert from one definition to the other.

   If the blackbody spectrum in terms of bandwidth is given as

   $$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$$

   then derive the blackbody spectrum in terms of wavelength. (Hint: consider $|B_\nu(T)\mathrm{d}\nu| = |B_\lambda(T)\mathrm{d}\lambda|$.)

2. Last week, we saw how faint most stars are in the radio part of the spectrum (i.e. in the frequency range $\nu \lesssim 10^{11}$ Hz).
   (a) Given that the peak of the blackbody spectrum for stars typically falls in the optical range ($\nu \approx 10^{15}$ Hz), derive the Wien approximation to the Planck spectrum given in the previous exercise. (Hint: Wien's approximation simplifies Planck's blackbody spectrum by moving the exponential to the numerator [i.e. get rid of the division so that the exponential ends up being merely a multiplicative factor].)
   (b) Now use the result from the previous question ($B_\nu(T) = 2h\nu^3/c^2 \exp(-h\nu/kT)$) to derive the Wien displacement law in terms of frequency (Note: last week we also looked at Wien's displacement law, but in terms of wavelength. After exercise 1, we can realise this is slightly different.)
   (c) Finally calculate the frequency at which the Solar radiation peaks (remember the Solar temperature we calculated last week, was $T = 5778\,\mathrm{K}$).

3. Earlier, we discussed how Kepler's law can be used to derive the masses of planets, assuming the mass of their moons are negligible in comparison. In the case of binary stars, this assumption is typically not correct. However, since both stars in such a system move symmetrically (weighted by their repective masses) around the common centre of gravity, it is easily realised that their mass ratio equals the inverse ratio of their orbital sizes: $m_1/m_2 = a_2/a_1$. Combining this equation with Kepler's third law ($P^2 M_{\mathrm{tot}} = a^3$, in which a is the sum of the semi-major axes of the two companion stars), we can disentangle the masses from both stars.

   Illustrate the above by calculating the masses for a binary pair of stars that orbit every 100 years and are at most $7''$ and at least $1''$ separated on the sky. Assume a distance (to Earth) of 10 pc; the heaviest star moves by $2''$ during its orbit.