Introduction to Astronomy
Exercises week 5

8 November 2019

1. The Solar flux at the Earth (remember $1\,\text{AU} = 1.495978 \times 10^{11}\,\text{m}$), is $1370\,\text{W/m}^2$. By integrating Planck's law (i.e. the blackbody spectrum), we can achieve Stefan-Boltzmann's law, which states that the frequency-integrated power output per unit surface area of a blackbody scales with temperature as: $M = \sigma T^4$, where $\sigma = 5.6705 \times 10^{-8}\,\text{Wm}^{-2}\text{K}^{-4}$ is the Stefan-Boltzmann constant and M is the total energy flux per unit surface area and unit time.
   (a) Calculate the total power output of the Sun and the Sun's temperature, assuming it is a blackbody. (Note the Solar radius is about 696,342 km.)
   (b) By differentiating Planck's law, we can find the frequency at which the luminosity of a blackbody peaks. This is called Wien's law and states: $\lambda_{\text{peak}} = hc/(\beta k T)$ with $\beta = 4.96511$, $k = 1.380658 \times 10^{-23}\,\text{J/K}$ Boltzmann's constant and $h = 6.626 \times 10^{-34}\,\text{Js}$ Planck's constant. Now calculate the peak wavelength and frequency for the Solar spectrum.
   (c) Using the numbers above, estimate the amount of mass-loss this power output equates with, per second. At this rate, how long does the Sun survive? (The Sun's weight is approximately 2×10^{30} kg.)

2. The sensitivity of radio telescopes is often expressed in terms of the Jansky (Jy), where $1\,\text{Jy} = 10^{-26}\text{W/m}^2\text{/Hz}$. Assuming a sensitivity of 1 mJy for a system that has a bandwidth of 300 MHz centred at 1.4 GHz, how far can you place the Sun before it becomes undetectable? Use the Rayleigh-Jeans approximation to the Planck spectrum, for long wavelengths: $I_\nu = 2kT\nu^2/c^2$. Given that $1\,\text{AU} = 4.85\,\mu\text{pc}$ and with a Galactic radius of $\sim 15\,\text{kpc}$, in what fraction of the Galaxy can we see Sun-like stars in the radio?