Introduction to Astronomy
Exercises Week 3

25 October 2019

1. When the Earth is in perigee (147098 Mm from the Sun), the Sun has an angular diameter of 32.5′. Assuming the Sun is spherical, and using Kepler's third law $T^2/r^3 = 4\pi^2/(GM)$ with $G = 6.67259 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$ Newton's gravitational constant, determine the density of the Sun. (1 AU$= 1.496 \times 10^{11}$ m.)

2. For an interstellar cloud to gravitationally collapse, it must be beyond its *Jeans mass*, which is roughly defined as:
$$M_{\mathrm{Jeans,M}_\odot} \approx 3 \times 10^4 \sqrt{T_\mathrm{K}^3 / n_{\mathrm{atoms/m^3}}}.$$
For a typical interstellar molecular cloud, with a temperature of 20 K and a typical density of 1000 atoms/cm^3, calculate the required mass for gravitational collapse.

3. Using conservation of energy and conservation of angular momentum, derive the vis-viva relation (conservation of orbital energy):
$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right).$$