Introduction to Astronomy
Exercises Week 2

18 October 2019

1. The resolution of telescopes can be approximated as:

$$\theta \approx 1.22 \frac{\lambda}{D}$$

where θ is the resolution in radians, λ is the observing wavelength in metres and D is the aperture diameter in metres.

   (a) Considering the 10-m Keck telescopes operating at a wavelength of 580 nm (in the yellow part of the spectrum), calculate the minimum angular diameter (in radians and arcseconds) a source will have to be in order to be resolved.

   (b) With interferometric techniques, multiple telescopes can be used in combination to synthesise a large one. In that case the aperture diameter in the above equation becomes equal to the maximal spacing between telescopes.

   Calculate the resolution of the very large array – a 27-element interferometer in New Mexico – operating at an observing frequency of 50 GHz and in its most extended configuration, with a maximal baseline length of 36 km.

2. (a) The orbital period of Mars around the Sun is 1.8808 years. Using Kepler's third law ($a^3_{\mathrm{AU}} = (M_{1,M_\odot} + M_{2,M_\odot}) P^2_{\mathrm{yrs}}$), calculate the semi-major axis of the Martian orbit.

   (b) Using the result from the previous question and the fact that the astronomical unit equals 1.496×10^8 km, estimate the radius of Mars if its angular radius is observed to be $9.2''$ at opposition (when the Earth is directly between Mars and the Sun). Approximate orbits as perfectly circular.

   (c) Now that you know how far Mars is, you can calculate its mass by observing the orbit of its moon. Specifically, Phobos orbits Mars in 0.3189 days and when Mars is in opposition, the maximum separation between Phobos and Mars is seen to be $25''$. What is Mars' mass (compared to Earth)? (Assume a circular orbit for Phobos; and use a solar mass of $\mathrm{M}_\odot = 332968 \times \mathrm{M}_{\mathrm{Earth}}$.)

3. There are two obvious definitions of a day: the sidereal day, after which the Earth has regained its orientation with respect to the stars; and the Solar day, after which the Earth has regained its orientation with respect to the Sun. Calculate the difference between the two; and how much does this difference add up to in a year? (Note that there are 86400 seconds in a Solar day and 365.25 days in a year, *by definition*.)