Introduction to Astronomy – Exam Formula Sheet

Formulae

Note 1: log implies the logarithm base 10; whereas the natural logarithm (when needed) is denoted by ln.
Note 2: The units are not necessarily SI. Please use common sense to double-check in case of doubt.

- Kepler's third law: $P^2 M_{\rm tot} = a^3_{\rm comb}$
- Escape velocity: $v_{\rm esc} = \sqrt{\frac{2GM}{R}}$
- Hubble's law in terms of redshift: $z = HD/c$
- Definition of apparent magnitudes: $m = -2.5 \log F/F_{\rm ref}$
- Definition of absolute magnitude: $M = m - 5 \log \frac{D}{10\,{\rm pc}}$
- Definition of absolute magnitude accounting for interstellar extinction: $M = m - 5 \log \frac{D}{10\,{\rm pc}} - aD$
- Period-magnitude relation for Cepheid variables: $\langle M \rangle = -2.78 \log \frac{P}{10\,{\rm days}} - 4.13$
- Stefan-Boltzmann's law: $F = \sigma T^4$
- Moment of inertia for a homogeneous sphere: $I = \frac{2}{5} M R^2$
- Emission measure: ${\rm EM} = \int_0^d n_{\rm e}^2 {\rm d}l$
- Optical depth for Bremsstrahlung (free-free emission):

$$\tau_\nu = 8.235 \times 10^{-2} \left(\frac{T}{1\,{\rm K}}\right)^{-1.35} \left(\frac{\nu}{1\,{\rm GHz}}\right)^{-2.1} \frac{\rm EM}{1\,{\rm cm}^{-6}{\rm pc}}$$

- Jeans Mass:

$$\frac{M_{\rm Jeans}}{M_\odot} \approx 3 \times 10^4 \sqrt{\left(\frac{T}{1\,{\rm K}}\right)^3 \left(\frac{n_{\rm atoms}}{\rm m^{-3}}\right)^{-1}}.$$

- The vis-viva relation: $v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$
- Wien's law: $\lambda_{\rm peak} = \frac{hc}{\beta k T}$, with $\beta = 4.96511$
- Rayleigh-Jeans approximation: $I_\nu = 2kT\nu^2/c^2$.
- Planck spectrum:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(h\nu/kT\right) - 1}$$

- Wien's approximation:$B_\nu(T) = 2h\nu^3/c^2 \exp\left(-h\nu/kT\right)$.

Constants

- The absolute magnitude of the Sun: $M_\odot = 4.8$.
- The apparent magnitude of the Sun (in V band): $m_\odot = -26.74$.
- The luminosity of the Sun: $L_\odot = 3.846 \times 10^{26}\,\mathrm{W}$.
- The radius of the Sun: $R_\odot = 6.96 \times 10^{8}\,\mathrm{m}$.
- The mass of the Sun: $M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$.
- The orbital radius of the Moon: $D_{\mathrm{Moon}} = 3.84 \times 10^{8}\,\mathrm{m}$.
- The size of the AU: $1.49597870 \times 10^{11}\,\mathrm{m}$.
- The size of a parsec: $3.0857 \times 10^{16}\,\mathrm{m}$.
- The mass of an electron: $m_\mathrm{e} = 9.1 \times 10^{-31}\,\mathrm{kg}$.
- The charge of an electron: $q_\mathrm{e} = -1.6 \times 10^{-19}\,\mathrm{C}$.
- The mass of a proton: $m_\mathrm{p} = 1.67 \times 10^{-27}\,\mathrm{kg}$.
- The charge of a proton: $q_\mathrm{p} = 1.6 \times 10^{-19}\,\mathrm{C}$. (Note 1 C = 1 Ampere second.)
- The mass of a hydrogen atom: $M_\mathrm{H} = 1.67 \times 10^{-24}\,\mathrm{g}$.
- Planck's constant: $h = 6.626 \times 10^{-34}\,\mathrm{Js}$.
- Boltzmann's constant: $k = 1.380658 \times 10^{-23}\,\mathrm{J/K}$.
- Stefan-Boltzmann's constant: $\sigma = 5.6705 \times 10^{-8}\,\mathrm{W/m^2/K^4}$.
- The radiation constant: $a = 4\sigma/c = 7.5657 \times 10^{-16}\,\mathrm{J/m^3/K^4}$.
- Newton's gravitational constant: $G = 6.67259 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$.
- The Jansky: $1\,\mathrm{Jy} = 10^{-26}\,\mathrm{W/m^2/Hz}$.
- Hubble's constant: $68\,\mathrm{km/s/Mpc}$.