Quantum Mechanics: Exercises 8

Due to: December 18, 2012.

Problem 1 (Continuation of Problem 2 in set no. 7)

c) Show that a coherent state can also be obtained by applying the translation (finite-displacement) operator $e^{-i\hat{p}l/\hbar}$ (where $\hat{p}$ is the momentum operator and l is the displacement distance) to the ground state.
d) Show that a coherent state $|\lambda\rangle$ remains coherent under time-evolution and calculate time-evolved state $|\lambda(t)\rangle$.

Problem 2

Derive the quantum equation of motion for the operator of kinematic momentum.
Hint: First show that kinematic momentum can be identified with $\boldsymbol{\pi} = \boldsymbol{p} - e\boldsymbol{A}$. In the Heisenberg picture the equation of motion for $\boldsymbol{\pi}$ is given by

$$\dot{\boldsymbol{\pi}} = \frac{1}{i\hbar}[\boldsymbol{\pi}, H]. \tag{1}$$

Express Hamiltonian for the electromagnetic field in terms of $\boldsymbol{\pi}$ and use previous equation to find equation of motion.

Problem 3

Determine the energy spectrum and wave functions corresponding to a charged particle in uniform crossed electric and magnetic fields, with $\boldsymbol{B}$ in the z direction and $\boldsymbol{E}$ in the x direction.
Hint: This problem may be easy or difficult, depending upon the vector potential that is chosen.