Quantum Mechanics: Exercises 7

Due to: December 11, 2012.

Problem 1

Hamiltonian for a two-state system is given by

$$H = \begin{pmatrix} E_0 & -\alpha \\ -\alpha & E_0 \end{pmatrix} \tag{1}$$

in the basis $|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

a) Find the eigenvalues and eigenvectors of H.

b) At $t = 0$ the system is in the state

$$|\psi(0)\rangle = \frac{|1\rangle + i|2\rangle}{\sqrt{2}} \tag{2}$$

What are the probabilities that at some time $t > 0$ system is found in the states $|1\rangle$ and $|2\rangle$? What are the probabilities that the system is found in each of the two energy eigenstates?

Problem 2

A coherent state of a one-dimensional simple harmonic oscillator is defined to be an eigenstate of the (non-Hermitian) annihilation operator $\hat{a}$

$$\hat{a}|\lambda\rangle = \lambda|\lambda\rangle, \tag{3}$$

where λ is, in general, a complex number

a) Prove that

$$|\lambda\rangle = e^{-|\lambda|^2/2} e^{\lambda \hat{a}^\dagger} |0\rangle \tag{4}$$

is a normalized coherent state.

b) Prove the minimum uncertainty relation for such a state.

Problem 3