Quantum Mechanics: Exercises 6

Due to: December 4, 2012.

Problem 1

Consider spin of the electron in strong magnetic field $\mathbf{B_0}$ in z-direction. Add some small magnetic field $\mathbf{b}$ in x-direction. Calculate energy and corresponding eigenvectors of the system exactly and then up to second order in perturbation theory.

Hints:
a) Operator of energy of electron magnetic moment in external field is given by $H = \frac{e\hbar}{2m}\boldsymbol{\sigma} \cdot \boldsymbol{B}$, where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$

Problem 2

Consider unperturbed Hamiltonian H_0, which is time independent and some small time dependent perturbation $H'(t)$. Assume that unperturbed problem is exactly solvable and eigenvalues are given by

$$H_0|\psi_n\rangle = E_n|\psi_n\rangle. \tag{1}$$

Assuming

$$|\psi(t)\rangle = \sum_n c_n(t)e^{-iE_n t/\hbar}|\psi_n\rangle \tag{2}$$

show that the Schrödinger equation is equivalent to

$$\frac{dc_m(t)}{dt} = \frac{1}{i\hbar}\sum_n H'_{mn}(t)c_n(t)e^{i\omega_{mn}t}, \tag{3}$$

where

$$H'_{mn}(t) = \langle\psi_m|H'(t)|\psi_n\rangle \tag{4}$$

and

$$\omega_{mn} = (E_m - E_n)/\hbar \tag{5}$$

. To first order in H' solution of (3) is given by

$$c_m(t) = c_m(t_0) + \frac{1}{i\hbar}\sum_n \int_{t_0}^{t} dt' H'_{mn}(t')c_n(t_0)e^{i\omega_{mn}t'}, \tag{6}$$

Problem 3

Linear harmonic oscillator is in the ground state and in time $t = -\infty$ we turn on a weak homogeneous electric field with intensity given by

$$E(t) = E_0 e^{-\frac{t^2}{\tau^2}}, \tag{7}$$

Using results of previous problem calculate probability of finding LHO in the n-th excited state at time $t = \infty$ (i.e. calculate $P_n = |c_n|^2$).