Quantum Mechanics: Exercises 5

Due to: November 27, 2012.

Problem 1

Using $\hat{a}|0\rangle = 0$ find wave function corresponding to the state $|0\rangle$ in the coordinate representation. Find also wave function corresponding to the first excited state $|1\rangle$.

Problem 2

Consider spin of the electron in strong magnetic field $\mathbf{B_0}$ in z-direction. Add some small magnetic field $\mathbf{b}$ in x-direction. Calculate energy and corresponding eigenvectors of the system exactly and then up to second order in perturbation theory.

Hints:
a) Operator of energy of electron magnetic moment in external field is given by $H = \frac{e\hbar}{2m}\boldsymbol{\sigma} \cdot \boldsymbol{B}$.

Problem 3

Wave function of the hydrogen atom in the ground state is

$$\psi_0 = \frac{1}{\sqrt{\pi}} a_0^{-\frac{3}{2}} e^{-\frac{r}{a_0}}, \tag{1}$$

where $a_0 = \hbar^2/m_e e^2 = 0.529 \times 10^{-10}$m. Calculate the shift in atomic energy levels due to the finite size of the nucleus.

Hints:
a) Assume that the charge of the proton is distributed uniformly through-out the volume of a sphere of radius $R = 10^{-15}$m.
b) First find the potential energy of the electron at the distance r from the center of the nucleus. Be careful in a case $r < R$.