Quantum Mechanics: Exercises 4

Due to: November 20, 2012.

Problem 1

a) Show that the commutation relation for momentum and position

$$[X, P] = i\hbar, \tag{1}$$

cannot be satisfied for any finite-dimensional matrices X and P.
b) Show that it is possible for *infinite-dimensional* matrices, for example

$$X = \sqrt{\frac{\hbar}{2m\omega}} \begin{pmatrix} 0 & \sqrt{1} & 0 & \dots \\ \sqrt{1} & 0 & \sqrt{2} & \dots \\ 0 & \sqrt{2} & 0 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \qquad P = \sqrt{\frac{\hbar m\omega}{2}} \begin{pmatrix} 0 & -i\sqrt{1} & 0 & \dots \\ i\sqrt{1} & 0 & -i\sqrt{2} & \dots \\ 0 & i\sqrt{2} & 0 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \tag{2}$$

Problem 2

Show that matrices for the position and momentum operators, X and P, in the basis formed by the energy eigenvectors of the harmonic oscillator are given by the matrices in Problem 1.

Problem 3

Find an expression for the expectation value of x^4 , in the n-th excited state of the harmonic oscillator.

Problem 4

In case of harmonic oscillator show that so-called *virial theorem* holds , i.e. $\langle T\rangle = \langle V\rangle = E/2$, where T is kinetic energy, V is potential energy and E is total energy.