Quantum Mechanics: Exercises 3

Due to: November 13, 2012.

Problem 1

The Hamiltonian for a two-state system is given by

$$H = a(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|), \tag{1}$$

where a is some number with dimension of energy. Find the energy eigenvalues and the corresponding energy eigenkets (as linear combination of $|1\rangle$ and $|2\rangle$).

Problem 2

Show that the commutator of the momentum operator with a function of the position operator is given by

$$[f(x),\hat{p}] = i\hbar \frac{\partial f}{\partial x} \tag{2}$$

Problem 3

Consider double δ -well with potential

$$V(x) = -\alpha[\delta(x-a) + \delta(x+a)]$$
(3)

Calculate the spectrum of bound states of this potential. Show how the spectrum depends on the distance a.