Quantum Mechanics: Exercises 2

Due to: November 6, 2012.

Problem 1

Show that a commutator of two Hermitian operators is anti-Hermitian, that is, it satisfies $\hat{A}^\dagger = -\hat{A}$.

Problem 2

Prove that

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}} e^{\hat{B}} e^{-\frac{1}{2}[\hat{A},\hat{B}]}, \tag{1}$$

provided the operators $\hat{A}$ and $\hat{B}$ satisfy $[\hat{A}, [\hat{A}, \hat{B}]] = [\hat{B}, [\hat{A}, \hat{B}]] = 0$.

Problem 3

One-dimensional Gaussian wawe packet function is given by

$$\psi(x) = \frac{1}{\pi^{\frac{1}{4}}\sqrt{d}} \exp\left[ikx - \frac{x^2}{2d^2}\right], \tag{2}$$

a) calculate expectation values of operators $\hat{x}$ and $\hat{x}^2$.
b) Take operator

$$\hat{p} = -i\hbar \frac{d}{dx}, \tag{3}$$

and find expectation values of operators $\hat{p}$ and $\hat{p}^2$.
c) find dispersions σ_x and σ_p, where dispersion is defined as

$$\sigma_A^2 = \left\langle (\triangle A)^2 \right\rangle = \left\langle A^2 \right\rangle - \left\langle A \right\rangle^2. \tag{4}$$

c) What is the value of the uncertainty product $\sigma_x^2 \sigma_p^2$ in this case?