Quantum Mechanics: Exercises 12

Due to: January 29, 2013.

Problem 1

Consider a degenerate gas of free electrons at zero temperature. We turn on homogeneous magnetic Field $\boldsymbol{B}$, which cause a shift of energy levels depending on spin orientation of electrons. The Hamiltonian of this interaction is given by

$$H_{int} = -\mu_0 \boldsymbol{\sigma} \cdot \boldsymbol{B}. \tag{1}$$

a) Write down equations, which determines Fermi energies for electrons with spin oriented in the direction of external magnetic field and for electrons with spins oriented in opposite direction.

b) Solve these equation in a case that external magnetic field is weak. Find relation between magnetisation of electron gas and external magnetic field and prove that magnetic susceptibility is given by

$$\chi = \frac{3}{2}\mu_0^2 n \frac{2m}{\hbar^2 k_F^2}, \tag{2}$$

where n is the number density of the electron gas and $\hbar k_F$ is the Fermi momentum.

Problem 2

System of identical particles is described by the Hamiltonian

$$H = \epsilon a^\dagger a + \lambda(a^\dagger + a), \qquad \epsilon > 0, \lambda \in \mathbb{R}, \tag{3}$$

where we have only one independent creation and annihilation operator, $a^\dagger$ and a. Find spectrum of Hamiltonian in a case that $a^\dagger$ is the creation operator for the
a) boson,
b) fermion.
Show that in the limit $\lambda \ll \epsilon$, energy of the ground state is the same in both fermion and boson case.