Quantum Mechanics: Exercises 11

Due to: January 22, 2013.

Problem 1

An observation of a three-particle system revealed that one of these particles is in the state described by the wave function $\varphi_1(\boldsymbol{x})$, another one in the state $\varphi_2(\boldsymbol{x})$, and the last one in the state $\varphi_3(\boldsymbol{x})$. Write down the total wave function of the system assuming that: (i) the three particles are identical bosons; (ii) the three particles are identical fermions; (iii) two of the particles are identical fermions and the third particle is a boson. (In this problem, neglect spin, that is, assume that the state of the particles is fully described by the wave function.)

Problem 2

N identical spin-$\frac{1}{2}$ particles are subjected to a one-dimensional simple harmonic oscillator potential. Assuming that the particles do not interact with each other, what is the ground state energy of this system? What is the highest occupied energy level?

Problem 3

Determine approximately the energy and wave function of the ground state of a helium atom. First, explain why the total spin of the electrons in the ground state is zero. Then, carry out a variational calculation of the ground state using

$$\phi(r_1, r_2) = \frac{Z^3}{\pi a^3} e^{-Z(r_1+r_2)/a} \tag{1}$$

as a trial wave function, where a is the Bohr radius and Z is taken as a variational parameter. If you need a hint, consult page 93 of the lecture notes.

Problem 4

Two identical spin-$\frac{1}{2}$ fermions move in one dimension in an infinitely deep potential well defined by $V(x) = 0$ for $x \in [-a, +a]$ and $V(x) = +\infty$ otherwise. Write down the ground state wave function and energy assuming that the total spin state of the fermions is a: (i) singlet; (ii) triplet. In both cases, discuss using perturbation theory how the ground state is affected by a short-range attractive interaction $V(x) = -\lambda\delta(x_1 - x_2)$ with $\lambda > 0$.