Seminar "Mathematical Physics"

Here is a list of possible topics for the talks. Each talk should be about 60 minute long and followed by a question and discussion session.

Variational principles (N.B., May 7): Give a mathematical introduction to variational calculus and give physical examples from classical mechanics and field theory. Suggested references: [Kna17,Kna18 section 8]

Noether Theorem (S.H., May 14): State and prove the Noether Theorem regarding continuous symmetries and conserved quantities (in both the classical and field theoretical version). Explain why this theorem is so important in physics. Suggested reference: [Tao06, sec. 1.4]

Introduction to Lie algebras (M.Sc., May 21): Give an introduction to the theory of Lie algebra over the field of complex numbers. Give a formal definition of an "abstract" Lie algebra and illustrate it with various examples ($\mathfrak{gl}(n)$, $\mathfrak{sl}(n)$ etc.). Define the notion of simple and semi-simple Lie algebras. Explain Cartan's criterion for semi-simplicity. Suggested references: [EW06] and [Tao13].

Classification of Cartan–Killing (M.Sp., May 28): Give an overview of the classification of the simple complex Lie algberas in terms of root systems and Dynkin diagrams. Suggested references: [EW06] and [Tao13].

Introduction to Lie groups (P.N., June 4): Give the formal definition of a Lie group and prove that every matrix Lie group (i.e. a closed subgroup of $\mathrm{GL}(n, \mathbb{C})$) is an embedded submanifold of $\mathrm{GL}(n, \mathbb{C})$. Thus, it is a Lie group. For this purpose you should define the Lie algebra associated to a matrix Lie group and explain the relation with the previous talk. Suggested references: [Hal15, Chap. 1,2 and 3], [Bak02].

Introduction to Supersymmetry (B.M., June 11): Ask Tim!

Introduction to Random Matrix Theory (N.A., June 18): Give an introduction to Random Matrix Theory. Focus on the basic ideas to connect the notion of matrices to probability theory and introduce random matrix models such as GOE. Suggested references: [LNV17], [AGZ09], [EKR15].

Representation of groups Give an introduction to the representation theory of finite groups over the field of complex numbers. State and prove the theorem of Maschke and explain the classification of the irreducible representations using the theory of characters. The talk should be illustrated with small examples (cyclic groups, symmetric groups, etc.). Suggested references: [Ser77] or [FH91].

Young diagrams and representations of Symmetric groups Using the previous talk, explain that the irreducible representations of the symmetric group Sn are in bijection with partitions of the integer n. Introduce the notion of Young diagrams and Specht modules. Explain how the representation theory of the symmetric group is related to the combinatorics of Young diagrams (e.g. Frobenius character formula, Young's rule or Littlewood–Richardson rule). Suggested reference: [Ful97].

Spontaneous symmetry breaking Explain the concept of spontaneous symmetry breaking and state the Goldstone Theorem. Use examples such as spin systems and Higgs mechanism to illustrate the differences between explicit symmetry breaking, spontaneous symmetry breaking and anomal symmetry breaking. Suggested references: [Gol61], [Nam60], [Hig64], [vD12], [Wei96, sec. 19 and 21]

Introduction to Padé Approximants Suggested reference: [Zin71].

References:

[AGZ09] Greg W. Anderson, Alice Guionnet, and Ofer Zeitouni, *An Introduction to Random Matrices*, Cambridge University Press, 2009.

[Bak02] Andrew Baker, *Matrix groups. An introduction to Lie group theory*, Springer, London, 2002.

[EKR15] Bertrand Eynard, Taro Kimura, and Sylvain Ribault, *Random matrices*, 2015.

[EW06] Karin Erdmann and Mark J. Wildon, *Introduction to Lie algebras.* Springer, London, 2006.

[FH91] William Fulton and Joe Harris, *Representation theory. A first course* (Graduate Texts in Mathematics, vol. 129), Springer, New York, 1991.

[Ful97] William Fulton, *Young tableaux. With applications to representation theory and geometry* (London Mathematical Society Student Texts, vol. 35), Cambridge University Press, 1997.

[Gol61] J. Goldstone, *Field Theories with Superconductor Solutions*, Nuovo Cim. **19** (1961) 154–164.

[Hal15] Brian Hall, *Lie groups, Lie algebras, and representations. An elementary introduction*, (Graduate Texts in Mathematics, vol. 222), Springer, Heidelberg, 2015.

[Hig64] Peter W. Higgs, *Broken Symmetries and the Masses of Gauge Bosons*, Phys. Rev. Lett. **13** (1964) 508–509.

[Kna17] Andreas Knauf, *Mathematische Physik: Klassische Mechanik*, Springer-Verlag, 2017.

[Kna18] Andreas Knauf, *Mathematical Physics: Classical Mechanics*, Springer, 2018.

[LNV17] Giacomo Livan, Marcel Novaes, and Pierpaolo Vivo, *Introduction to Random Matrices – Theory and Practice*, Springer, 2017.

[Nam60] Yoichiro Nambu, *Quasiparticles and Gauge Invariance in the Theory of Superconductivity*, Phys. Rev. **117** (1960) 648–663.

[Ser77] Jean-Pierre Serre, *Linear representations of finite groups* (Graduate Texts in Mathematics, vol. 42), Springer, New York-Heidelberg, 1977.

[Tao06] Terence Tao, *Nonlinear dispersive equations: local and global analysis*, `http://www.math.ucla.edu/~tao/preprints/chapter.pdf`, (2006)

[Tao13] Terence Tao, *Notes on the classification of complex lie algebras*, `https://terrytao.wordpress.com/2013/04/27/notes-on-the-classification-of-complex-lie-algebras`, 2013.

[vD12] Suzanne van Dam, *Spontaneous symmetry breaking in the Higgs mechanism*, `http://philsci-archive.pitt.edu/9295/`, 2012.

[Wei96] Steven Weinberg, *The Quantum Theory of Fields, volume 2*, Cambridge University Press, 1996.

[Zin71] Jean Zinn-Justin, *Course on Padé Approximants*, in: *Strong Interaction Physics* (Springer Tracts in Modern Physics, vol. 57). Springer, Heidelberg, 1971.