Tutorial sheet 11

10. A model for depolarization: master equation

We consider further the model introduced in exercise **9.**, in which a two-level system $\mathcal{S}$ and a four-level environment $\mathcal{R}$ interact together.

i. Lindblad equation

Starting from the set of Kraus operators determined in exercise **9.**, show that the reduced statistical operator $\hat{\rho}_{\mathcal{S}}$ of the system $\mathcal{S}$ obeys a master equation of the Lindblad form

$$\frac{\mathrm{d}\hat{\rho}_{\mathcal{S}}(t)}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}\big[\hat{\mathsf{H}}_{\mathrm{eff.}}, \hat{\rho}_{\mathcal{S}}(t)\big] - \Gamma\bigg(\hat{\rho}_{\mathcal{S}}(t) - \frac{1}{2}\hat{\mathbb{1}}_{\mathscr{H}_{\mathcal{S}}}\bigg). \tag{1}$$

How is the rate Γ related to the probability p of exercise **9.**?

ii. Up to an irrelevant term proportional to the identity, the most general 2×2 Hermitian matrix is

$$\hat{\mathsf{H}}_{\mathrm{eff.}} = \frac{\hbar\omega}{2}\,\vec{n}\cdot\hat{\vec{\sigma}}, \tag{2}$$

where $\vec{n}$ is a unit vector. Using this form of $\hat{\mathsf{H}}_{\mathrm{eff.}}$ and the Bloch parameterization

$$\hat{\rho}_{\mathcal{S}}(t) = \frac{1}{2}\big[\hat{\mathbb{1}}_{\mathscr{H}_{\mathcal{S}}} + \vec{b}(t)\cdot\hat{\vec{\sigma}}\big], \tag{3}$$

show that the master equation (1) can be rewritten as

$$\frac{\mathrm{d}\vec{b}(t)}{\mathrm{d}t} = \omega\,\vec{n}\times\vec{b}(t) - \Gamma\vec{b}(t). \tag{4}$$