Tutorial sheet 2

Discussion topic: What are "Markovian" "linear" out-of-equilibrium processes? Which properties obey the kinetic coefficients entering the fundamental relationships in such processes?

3. Conjugate variables in a moving fluid

Determine the intensive variable conjugate to particle number density in a moving fluid.

Hint: Remember the derivation of the intensive variable conjugate to momentum.

4. Heat transport in a rod

Consider a linear heat conducting bar of length ℓ connecting two heat reservoirs at temperatures $T^{(A)}$ and $T^{(B)}$ with $T^{(A)} > T^{(B)}$. The entire system is thermally insulated from the outside world. We assume the heat flux to be small enough for the reservoirs to remain at constant temperatures and we consider the stationary (but not equilibrated!) situation $\partial T/\partial t = 0$.

i. Recall the expression of Fourier's law for heat transport and deduce the equation satisfied by the local temperature in the rod. Give the solution to this equation fulfilling the boundary conditions of the problem.

ii. Entropy production

Let Q be the heat transferred per unit time from A to B and $\mathcal{S}$ the cross-sectional area of the bar.

a) Give the energy flux density flowing along the rod, as well as the corresponding entropy flux density.

b) Consider a section $[x, x + \mathrm{d}x]$ of the rod. Write down the net entropy balance in the slice due to entropy flux entering and leaving it in the general case.

c) In the stationary situation considered here, the entropy in the slice remains by definition constant in time. Write down the local balance equation for entropy and deduce how much entropy is created at each point in the rod. Give the rate of entropy production over the whole rod. What do you recognize?

5. Relation between viscosity and diffusion

Consider very small particles (but macroscopic compared to atomic scales) of mass m suspended in a fluid in thermal equilibrium at temperature T. The particle density as a function of height z is $n(z)$ and g denotes the gravitational acceleration.

1. Show that $n(z)$ has the form $n(z) = n_0\,\mathrm{e}^{-\lambda z}$. Determine λ as a function of m, g, k_B and T. If we want to have observable effects over distances of the order of a centimeter at $T = 300$ K, what should be the order of magnitude of the mass m?

2. The particles are now under the influence both of gravity and a friction force proportional to velocity $\vec{F} = -6\pi R\eta\vec{v}$, with η the fluid shear viscosity and R is the radius of the particles, assumed to be spherical. What is the limiting velocity $v_{\rm lim}$ of the particles in the gravitational field?

3. The particles are subjected to two mutually opposing influences: on the one hand, they move down with a velocity $v_{\rm lim}$, on the other hand, diffusion tries to re-establish a uniform density. Let D be the diffusion coefficient of the particles in the fluid. What is the diffusion flux density $\vec{\mathcal{J}}_N$? Why is it directed toward $z > 0$?

4. By considering that at equilibrium the gravitational and diffusion effects balance each other, establish the (Stokes–Einstein) relation between the shear viscosity and the diffusion coefficient.