Tutorial sheet 1

Discussion topics:

– What are “affinities” and “fluxes” in out-of-equilibrium systems? How do they relate to entropy production?
– How is local thermodynamic equilibrium defined?

1. Flow of a gas between two containers at different temperatures and pressures

Consider a composite system made of two containers A and B with a classical ideal monoatomic gas of particles of mass m at temperatures $T^{(A)}$, $T^{(B)}$ and pressures $\mathcal{P}^{(A)}$, $\mathcal{P}^{(B)}$ respectively. Let $\Delta T \equiv T^{(B)} - T^{(A)}$ and $\Delta\mathcal{P} \equiv \mathcal{P}^{(B)} - \mathcal{P}^{(A)}$ denote the temperature and pressure differences.

i. Recall the expression of the Maxwell–Boltzmann distribution $p(\vec{v})$ for the velocities in an ideal gas at temperature T. Assuming that the particle density is uniform, write down the number density $f(\vec{r},\vec{v})\,\mathrm{d}^3\vec{v}$ of particles per unit volume with a velocity between $\vec{v}$ and $\vec{v}+\mathrm{d}^3\vec{v}$.

ii. Particle flow

A small hole of cross section $\mathcal{S}$ in the wall separating the containers allows gas to slowly flow from one container to the other. Where are at time t the gas particles which will traverse the hole with a given velocity $\vec{v}$ between t and $t+\mathrm{d}t$? Show that the number of particles flowing from container A to container B per unit time is

$$\mathcal{J}_N^{(A)} = \frac{\mathcal{P}^{(A)}\mathcal{S}}{\sqrt{2\pi m k_B T^{(A)}}}.$$

Deduce the overall particle flux $\mathcal{J}_N \equiv \mathcal{J}_N^{(A)} - \mathcal{J}_N^{(B)}$ and express it as a function of ΔT and $\Delta\mathcal{P}$, assuming those differences are small.

iii. Energy flow

Show that the flow of (kinetic) energy through the hole from container A to container B per unit time is

$$\mathcal{J}_E^{(A)} = \sqrt{\frac{2k_BT^{(A)}}{\pi m}}\,\mathcal{P}^{(A)}\mathcal{S}.$$

Deduce the overall energy flux $\mathcal{J}_E \equiv \mathcal{J}_E^{(A)} - \mathcal{J}_E^{(B)}$ and express it as a function of ΔT and $\Delta\mathcal{P}$.

iv. The chemical potential of a classical ideal gas is given by

$$\mu(T,\mathcal{V},N) = -k_BT\ln\left[\frac{\mathcal{V}}{N}\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}\right].$$

Express $\mathcal{J}_N$ and $\mathcal{J}_E$ as a function of the differences (beware the signs!)

$$\Delta\!\left(\frac{1}{T}\right) \equiv \frac{1}{T^{(B)}} - \frac{1}{T^{(A)}} \quad\text{and}\quad \Delta\!\left(-\frac{\mu}{T}\right) \equiv \frac{\mu^{(A)}}{T^{(A)}} - \frac{\mu^{(B)}}{T^{(B)}}.$$

What do you recognize?

2. Local thermodynamics in a simple fluid — fundamental equation for the densities

Show that in a simple fluid the entropy density s is related to the local thermodynamic densities χ_a and to their conjugate intensive variables Υ_a by

$$\sum_j{}' \Upsilon_a \chi_a = s - \frac{\mathcal{P}}{T},$$

with $\mathcal{P}$ the pressure and T the temperature, where the sum runs over energy density, particle number density and the components of momentum density. Which known relation do you recognize?