Tutorial sheet 3

Reminder on equilibrium statistical mechanics: refresh your knowledge on the *phase space density* of a classical many-particle system and on the quantum mechanical *density operator*.

5. Entropy production in a simple fluid

Consider a simple fluid, described by the densities of energy, particle number, and momentum. The corresponding intensive parameters and fluxes have been determined in the lecture. Here, we wish to investigate the entropy production in the fluid.

i. In the local rest frame comoving with the fluid at velocity $\vec{v}$ at some point, the energy flux density reduces to the flux of internal energy $\vec{\mathcal{J}}_U$. In analogy with Eq. (I.22) in the lecture notes, the entropy flux in this comoving rest frame is defined as

$$\vec{\mathcal{J}}'_S \equiv \frac{1}{T}\,\vec{\mathcal{J}}_U.$$

Justify why the fluxes of particle number and of momentum play no role here.

ii. How does the entropy density s transform in a Galilean transformation? Deduce the entropy flux density $\vec{\mathcal{J}}_S$ in a fixed rest frame.

iii. Write down the entropy balance equation, first in the general case, then in the linear regime relevant for Newtonian fluids. Discuss the result.

6. Moments and cumulants of some common probability distributions

i. Poisson distribution

Let λ be a positive real number. The Poisson distribution with parameter λ is defined on the discrete range of natural numbers $n \in \mathbb{N}$ by

$$p_n = \frac{\lambda^n}{n!}\,\mathrm{e}^{-\lambda}.$$

Find its first two moments, as well as its cumulants.

ii. Gaussian distribution

Let X be a continuous random variable, whose sample space Ω is $\mathbb{R}$, and μ, σ two real numbers. Give the first two moments and the cumulants of arbitrary order of the Gaussian probability distribution

$$p_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\,\exp\!\left[-\frac{(x-\mu)^2}{2\sigma^2}\right].$$

Find the moments of order greater than 2 in the case $\mu = 0$.