Elements of fluid dynamics for the modeling of heavy-ion collisions

Nicolas BORGHINI

Universität Bielefeld

Elements of fluid dynamics for the modeling of heavy-ion collisions

Lecture II:

- Overview on the dynamics of relativistic perfect fluids

- Application of perfect relativistic fluid dynamics to the description of high-energy nucleus-nucleus collisions

phenomenology & experiment

  - modeling of the bulk of the matter created in the collisions
  - yields a surprisingly good rendering of some measurements
  - ... at the cost of introducing elements that are not within perfect hydrodynamics

- Beyond perfect relativistic fluid dynamics

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Fluid dynamics & heavy-ion collisions

Why use fluid dynamics?

- … because Fermi (1950) & Landau (1953-56) said we could / should!

(at least, they told it for high-energy hadronic collisions)

570

Progress of Theoretical Physics, Vol. 5, No. 4, July~August, 1950

High Energy Nuclear Events

Enrico Fermi

Institute for Nuclear Studies
University of Chicago
Chicago, Illinois

(Received June 30, 1950)

Abstract

A statistical method for computing high energy collisions of protons with multiple production of particles is discussed. The method consists in assuming that as a result of fairly strong interactions between nucleons and mesons the probabilities of formation of the various possible numbers of particles are determined essentially by the statistical weights of the various possibilities.

Fluid dynamics & heavy-ion collisions

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COLLECTED PAPERS OF L. D. LANDAU

74. ON MULTIPLE PRODUCTION OF PARTICLES DURING COLLISIONS OF FAST PARTICLES

= Izv. Akad. Nauk. USSR **17** (1953) 51

1. GENERAL RELATIONS

Collisions of ultra-fast nuclear particles can be accompanied by the appearance of a large number of new particles (many-pronged stars in cosmic radiation). Fermi[1] propounded the ingenious idea of the possibility of applying

Fluid dynamics & heavy-ion collisions

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3. Distribution of Particles Produced in Energy and Direction

A study of the angular distribution of the particles formed, and their distribution in energy, requires a detailed consideration of the hydrodynamical motion of the matter in the system.

The relativistic hydrodynamic equations are contained in the relations

$$\frac{\partial T^{ik}}{\partial x^k} = 0, \qquad (7)$$

Fluid dynamics & heavy-ion collisions

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SUPPLEMENTO AL VOLUME III, SERIE X
DEL NUOVO CIMENTO

N. 1, 1956
1° Semestre

Not a valid argument?

Hydrodynamic Theory of Multiple Production of Particles.

S. Z. BELEN'KJI and L. D. LANDAU

Institute of Physical Problems of the Academy of Sciences of the USSR - Moscow
Institute of Physics of the Academy of Sciences of the USSR - Moscow

CONTENTS. — **1.** Introduction. – **2.** Termodynamic relationships in the break-up. – **3.** Total number of particles. – **4.** Energy and angular distribution of particles. – **5.** Collisions of particles of different masses.

Fluid dynamics & heavy-ion collisions

Why use fluid dynamics?

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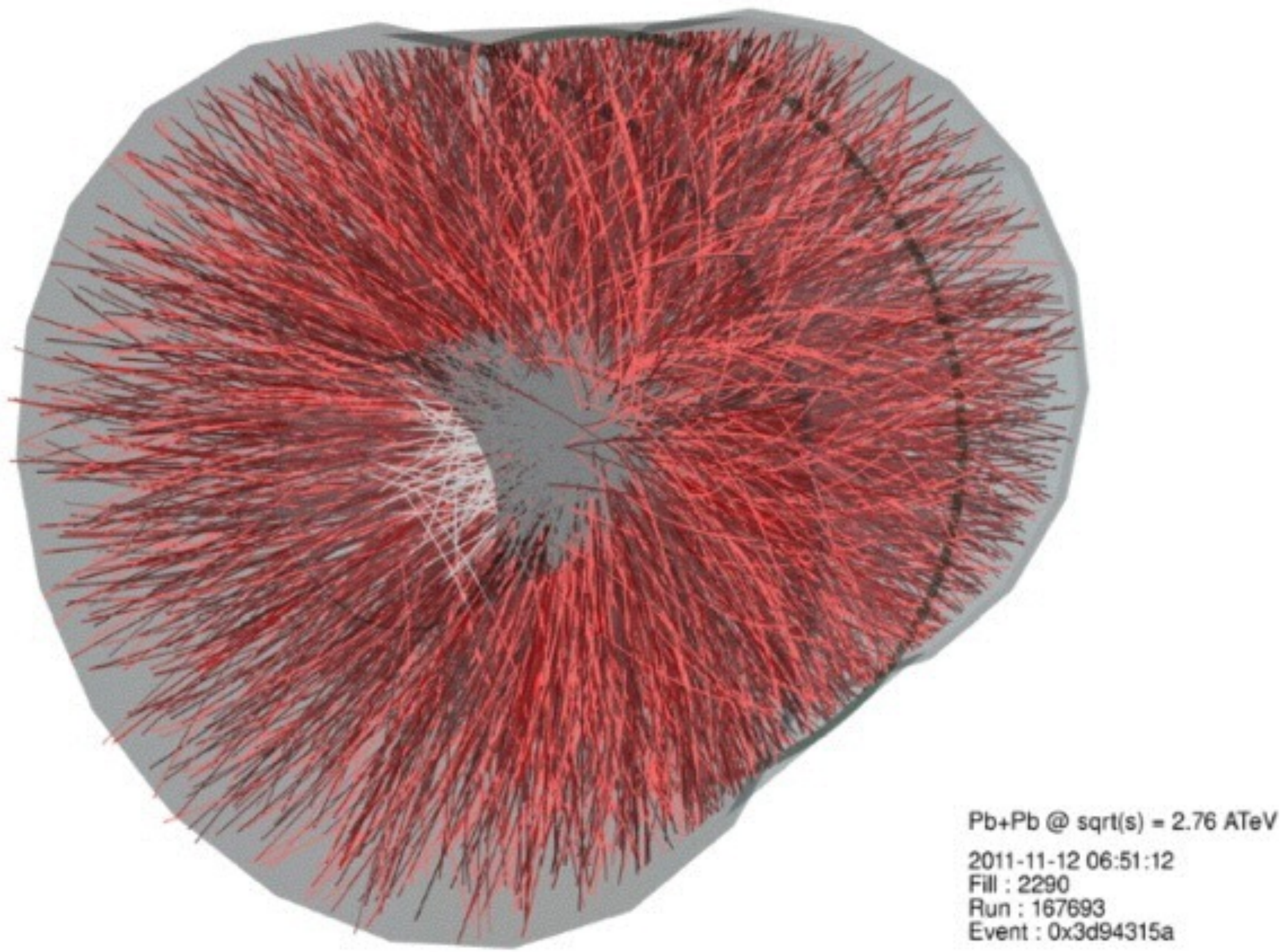
  (at least, they told it for high-energy hadronic collisions)
- … because you want to interpret that mess:



Fluid dynamics & heavy-ion collisions

Why use fluid dynamics?

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- … because you want to interpret that mess:

  ☛ a few thousands of particles per event at the LHC…

  Do you really want to use Pythia* to describe that?

  1. Till which version of Pythia do I have to wait?

     (How many free parameters will it have?)

  2. My computer may then understand… but shall I?

* or any other similar particle-based event generator

Fluid dynamics & heavy-ion collisions

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  ☛ a few thousands of particles per event at the LHC…

Statistical-physics based descriptions strike back!

☛ Fluid dynamics is not necessarily THE ultimate description — it has to break down in various regimes — yet it provides a useful reference and "easy" concepts.

Propaganda

Physics

Physics **3**, 105 (2010)

Viewpoint

A "Little Bang" arrives at the LHC

Edward Shuryak
Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794, USA

Published December 13, 2010

The first experiments to study the quark-gluon plasma at the LHC reveal that even at the hottest temperatures ever produced at a particle accelerator, this extreme state of matter remains the best example of an ideal liquid.

Subject Areas: **Particles and Fields**

A Viewpoint on:
Elliptic Flow of Charged Particles in Pb-Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV
K. Aamodt *et al.* (ALICE Collaboration)
Phys. Rev. Lett. **105**, 252302 (2010) – Published December 13, 2010

work made in NIKHEF!

Observation of a Centrality-Dependent Dijet Asymmetry in Lead-Lead Collisions at $\sqrt{s_{NN}} = 2.76$ TeV with the ATLAS Detector at the LHC
G. Aad *et al.* (ATLAS Collaboration)
Phys. Rev. Lett. **105**, 252303 (2010) – Published December 13, 2010

Space-time evolution of an ultrarelativistic heavy-ion collision

the classical view

- At $t = 0$, the Pb nuclei collide: "event" (10^5–10^7 events in a month run)
  - some of their internal constituents are stopped and set free from the nuclei wavefunctions ☞ QCD fields
  - at $t = 0^+$, the remnants of the nuclei fly away.
- First few fm/c: the liberated degrees of freedom form a "fireball"
  - which rapidly expands and cools down: collective behavior;
  - whose content (relevant degrees of freedom) evolves.
- At $t \approx 10$–20 fm/c, the fireball stops behaving collectively, particles fly freely to the detectors.

☞ about 2–$10 \cdot 10^3$ particles per event

Space-time evolution of an ultrarelativistic heavy-ion collision

the classical view

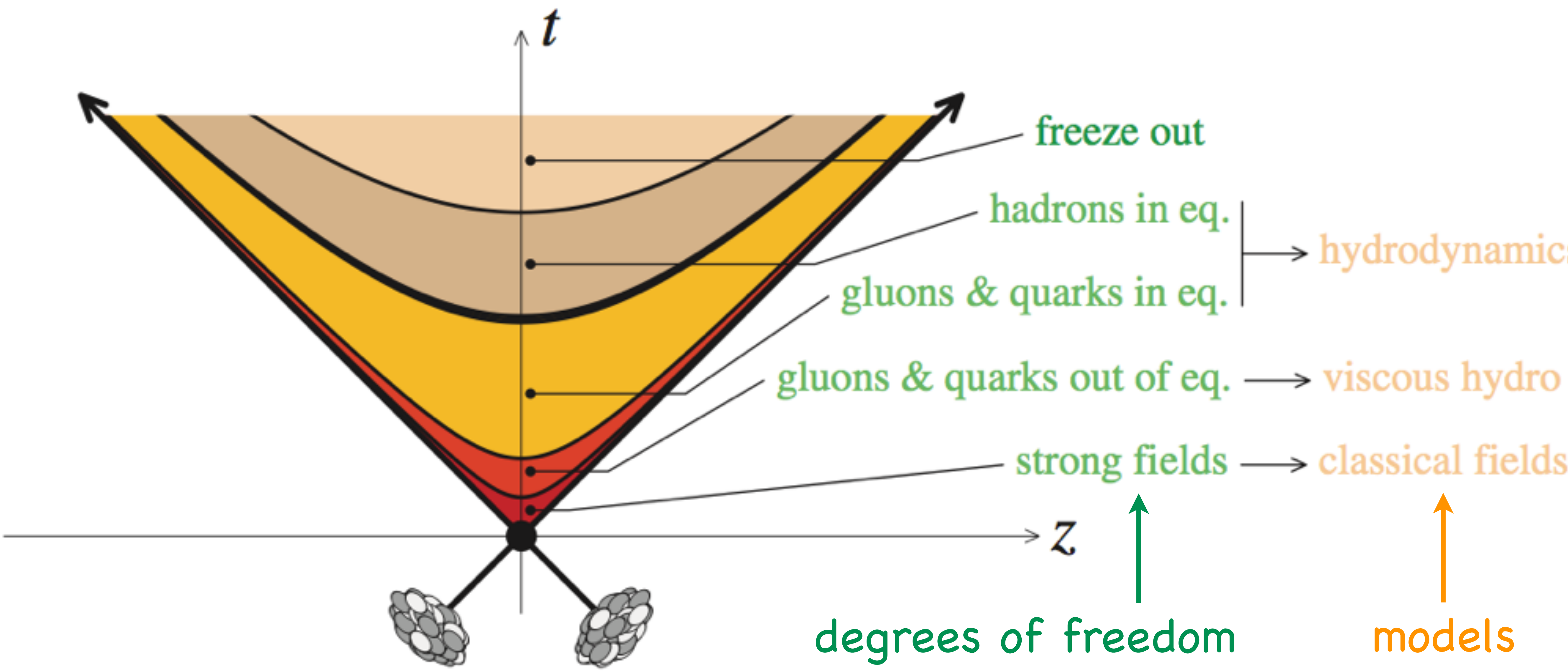


Figure taken from F.Gelis, Int. J. Mod. Phys. A **28** (2013) 133001

Fluid dynamics & heavy-ion collisions

modeling of the evolution

The usual picture of the space-time evolution of a nucleus-nucleus collision at ultrarelativistic energies (RHIC, LHC...) postulates the presence of a continuous medium in the final state.

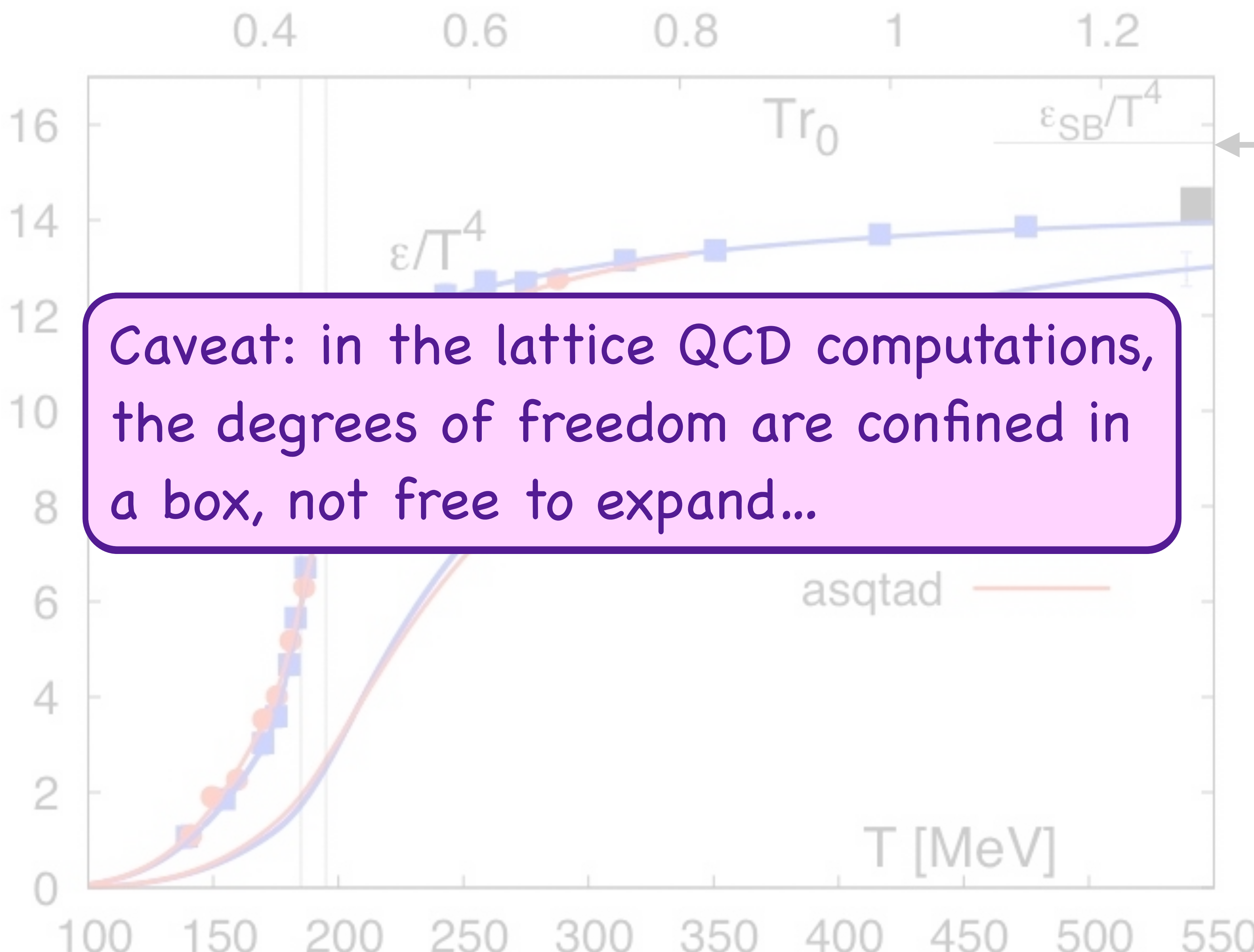
(Turn the argument round: the purpose of the experimental programs is to create and study this medium.)

Continuous medium: we know from Lecture I that at some point we need to worry about the Knudsen number.

Assuming now that everything is fine, what would be the properties of that medium? (if you believe what some theorists pretend)

Some results from lattice QCD

Energy density ϵ & pressure $\mathcal{P}$



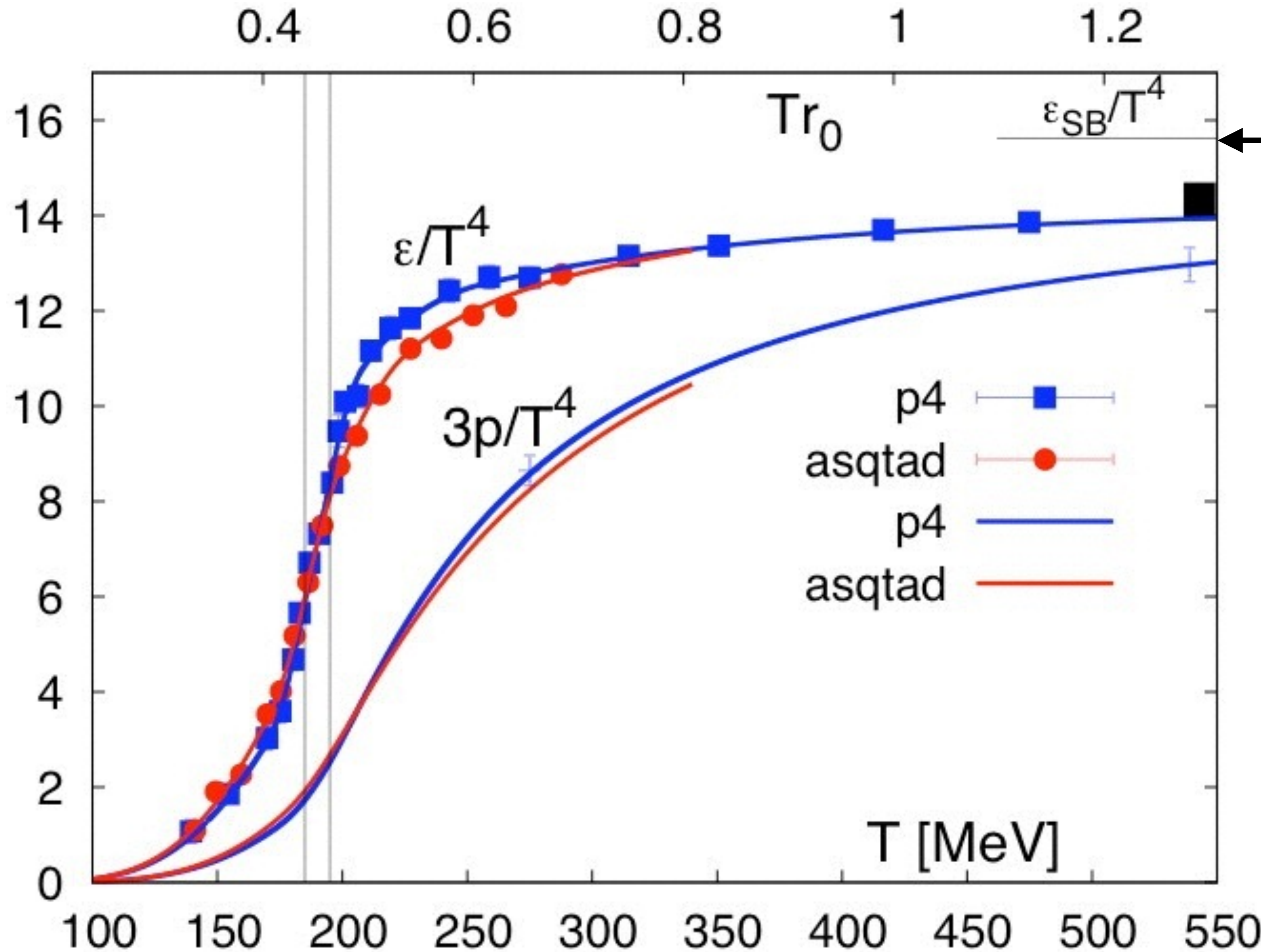
ideal gas limit

Caveat: in the lattice QCD computations, the degrees of freedom are confined in a box, not free to expand...

Plot from Bazavov et al., Phys. Rev. D **80** (2009) 014504

Some results from lattice QCD

Energy density ϵ & pressure $\mathcal{P}$

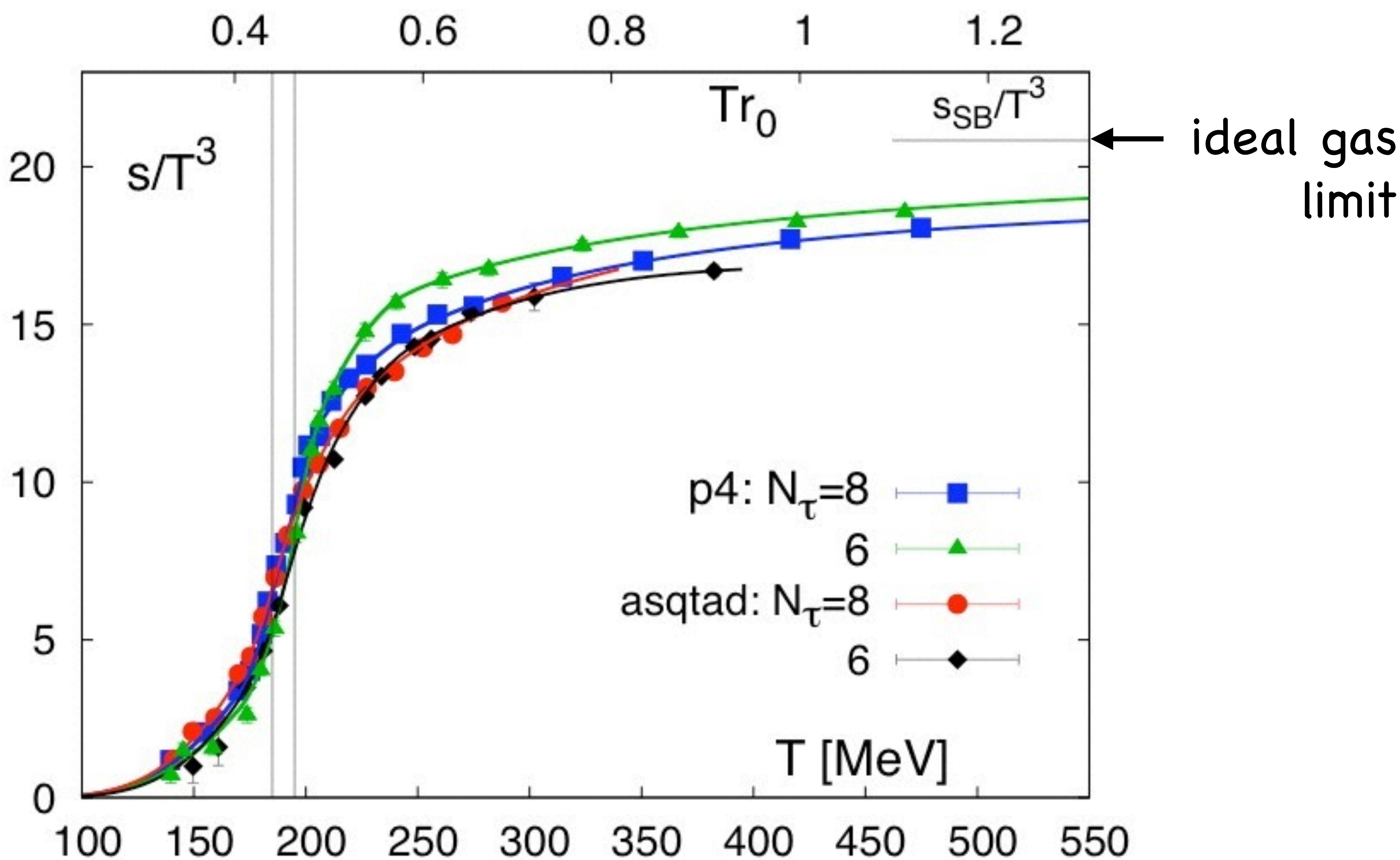


ideal gas limit

Plot from Bazavov et al., Phys. Rev. D **80** (2009) 014504

Some results from lattice QCD

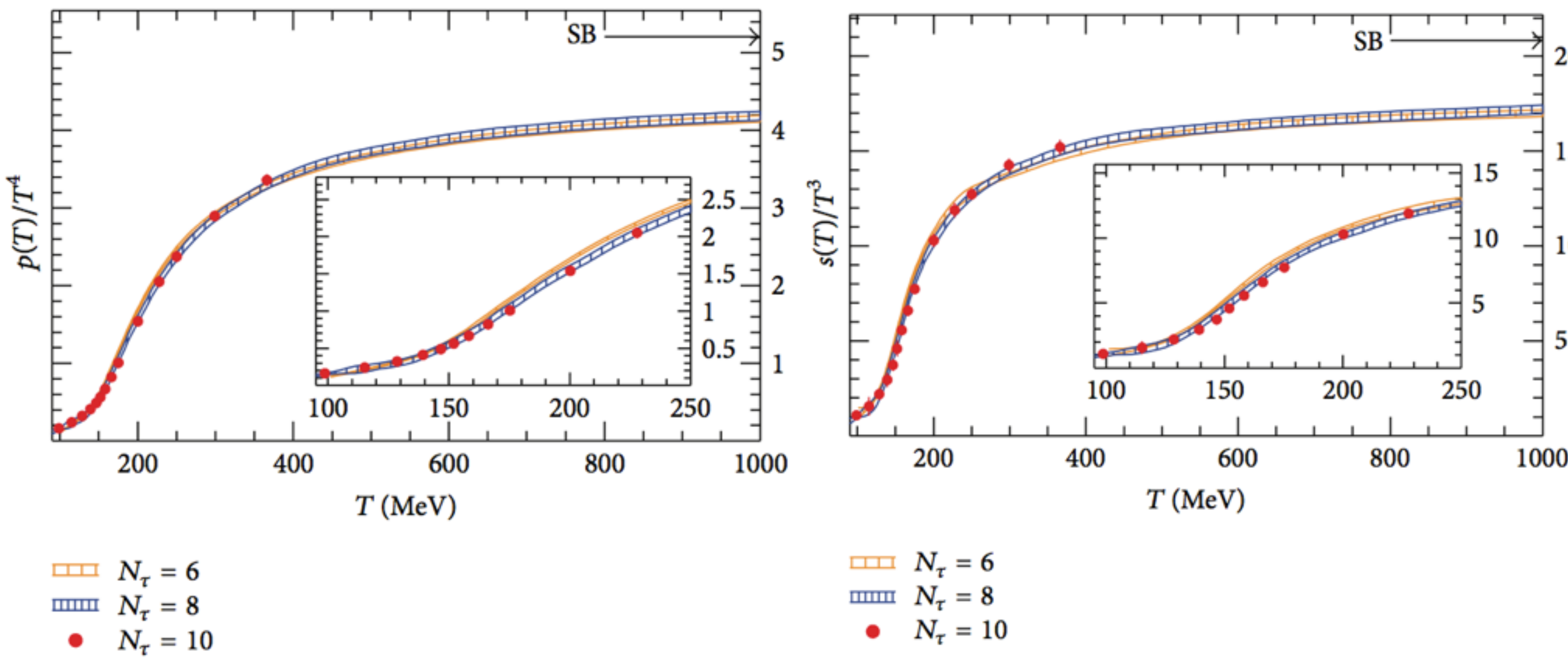
Entropy density s



Plot from Bazavov et al., Phys. Rev. D **80** (2009) 014504

Some results from lattice QCD

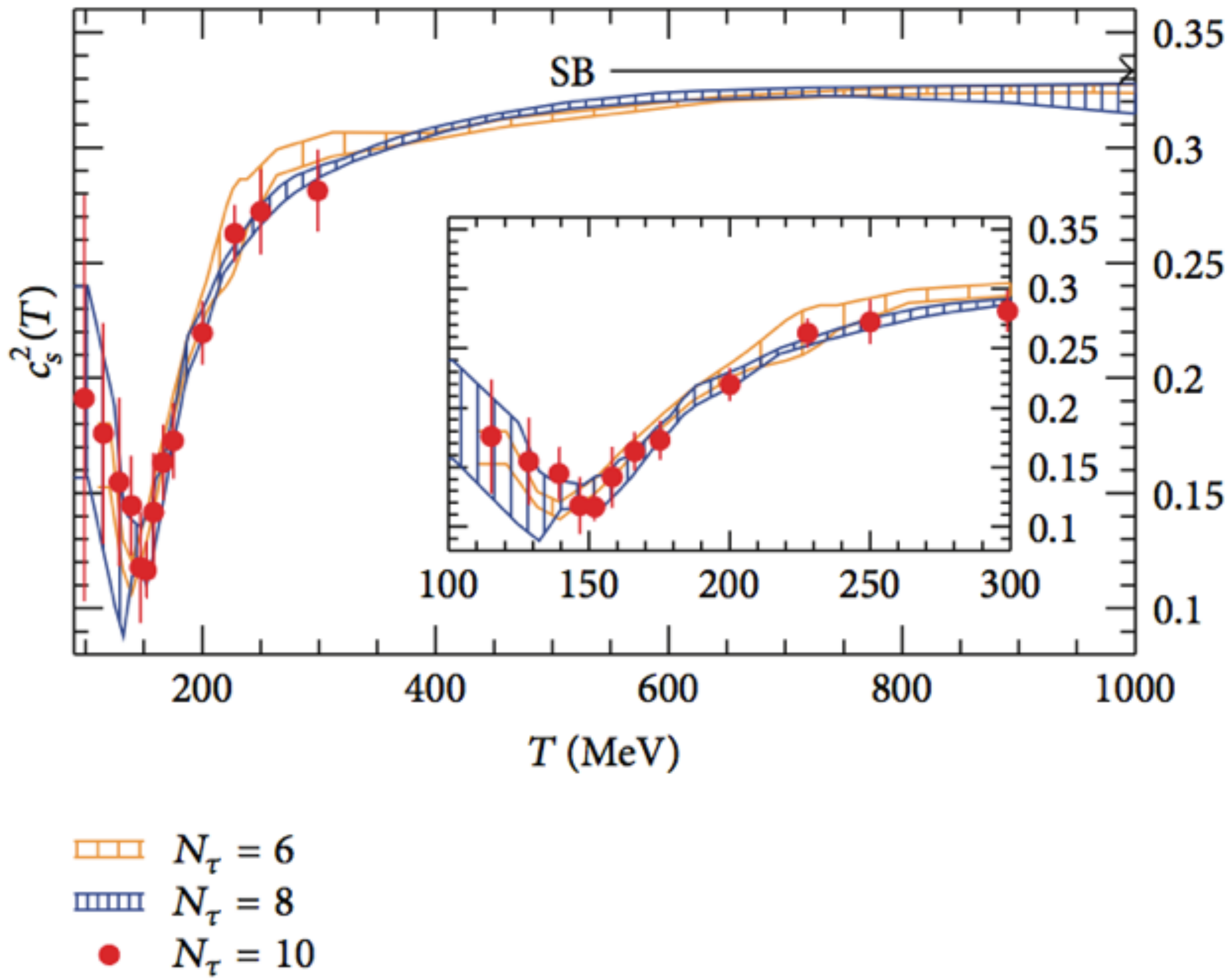
pressure $\mathcal{P}$ & entropy density s



Plot from [Borsanyi et al., JHEP 10 (2010) 077]

Some results from lattice QCD

speed of sound c_s: equation of state!



Plot from Borsanyi et al., JHEP 10 (2010) 077

Fluid dynamics & heavy-ion collisions

Lattice QCD computations provide "an"* equation of state for strongly interacting matter at high temperature — and vanishing baryon number density —, that may be used to model the fireball formed in a high-energy heavy-ion collision.

Performing fluid dynamical simulations — i.e. assuming that hydro is a valid model for the fireball evolution — with that equation of state and comparing their output to experimental results, one can test the EoS.

*up to the various choices of fermion implementation (number, masses, type), lattice size and plaquettes, extrapolation to the continuum limes...

Fluid dynamics & heavy-ion collisions

Performing fluid dynamical simulations — i.e. assuming that hydro is a valid model for the fireball evolution — with that equation of state and comparing their output to experimental results, one can test the EoS found in lattice QCD computations.

☞ **Constraining the Eq. of State of Super-Hadronic Matter from Heavy-Ion Collisions**

Scott Pratt,[1] Evan Sangaline,[1] Paul Sorensen,[2] and Hui Wang[2]

[1] *Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory Michigan State University, East Lansing, MI 48824, USA*

[2] *Brookhaven National Laboratory, Upton, New York 11973, USA*

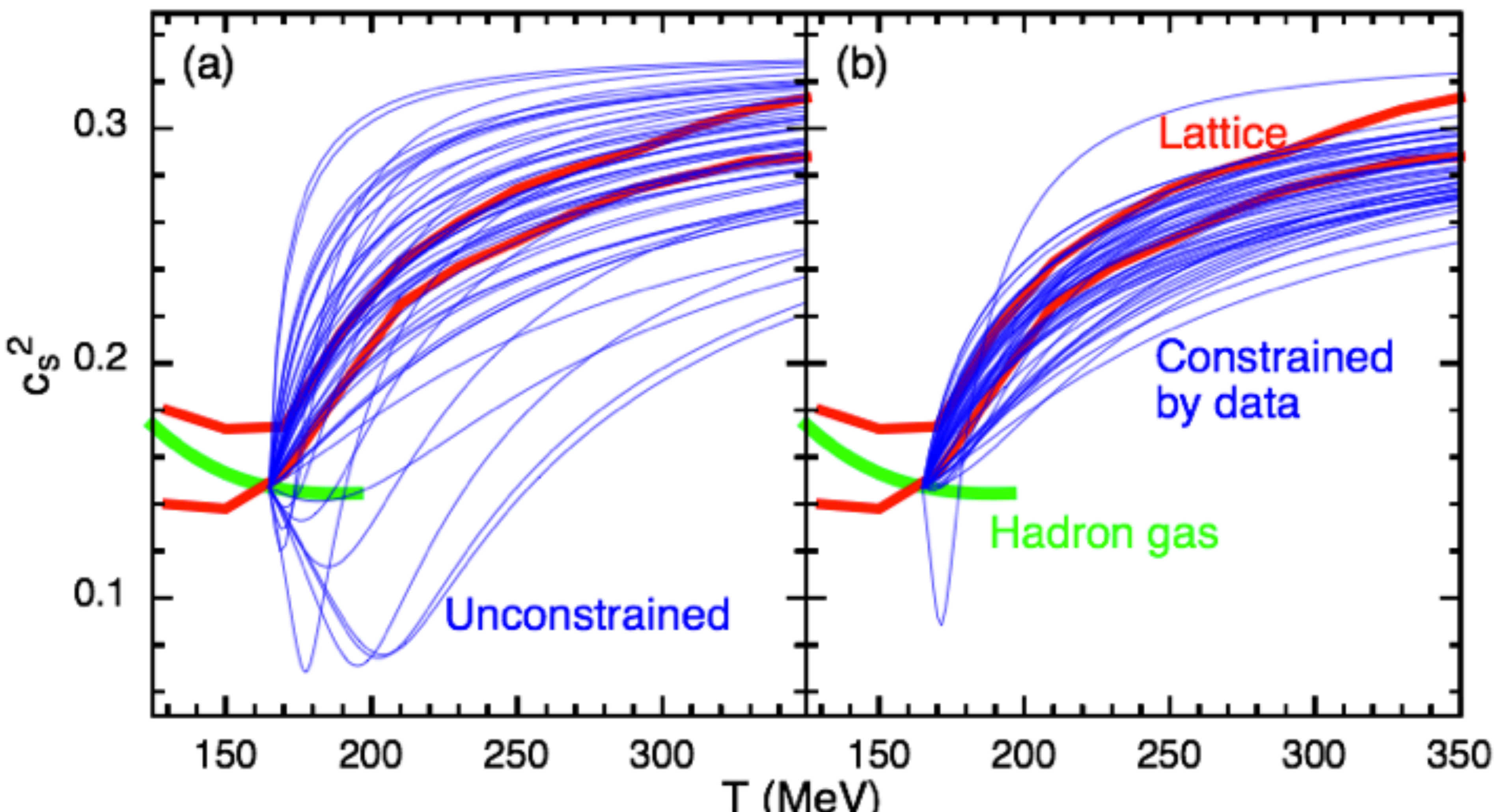
(Dated: January 19, 2015)

arXiv:1501.04042

The equation of state of QCD matter for temperatures near and above the quark-hadron transition (∼ 165 MeV) is inferred within a Bayesian framework through the comparison of data from the Relativistic Heavy Ion Collider and from the Large Hadron Collider to theoretical models. State-of-the-art statistical techniques are applied to simultaneously analyze multiple classes of observables while varying 14 independent model parameters. The resulting posterior distribution over possible equations of state is consistent with results from lattice gauge theory.

Fluid dynamics & heavy-ion collisions

Performing fluid dynamical simulations — i.e. assuming that hydro is a valid model for the fireball evolution — with that equation of state and comparing their output to experimental results, one can test the EoS found in lattice QCD computations.



plots from arXiv:1501.04042

Fluid dynamics & heavy-ion collisions

Phenomenologist's approach:

- The laws of relativistic fluid dynamics are known!
- The equation of state is more or less known.
- If using perfect fluid dynamics, you need nothing more:
  - perform your calculations (!!! here enter a few non-hydro items !!!)
  - compare with experimental data
  - publish (or perish)

Fluid dynamics & heavy-ion collisions

Phenomenologist's approach:

- The laws of relativistic fluid dynamics are known!

  but should one use perfect or dissipative (2nd order!) hydro?

- The equation of state is more or less known.

  improve on its experimental determination!

- If using dissipative fluid dynamics, you need transport coefficients:
  - assume their values (η, ζ, κ...): not (yet?) computed in lattice QCD
  - perform your calculations (!!! here enter a few non-hydro items !!!)
  - compare with experimental data
  - optimize your initial choice of transport coefficients and go back to the first step
  - publish (or perish)

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Fluid dynamics & heavy-ion collisions

scenarios

- Longitudinal (= along the colliding beam direction) expansion
  - Landau scenario
  - Bjorken scenario
- Transverse expansion
  - "transverse collective flow"
  - in particular "anisotropic flow"
- Later stages
  - conversion into particles ("freeze-out")

Fluid dynamics & heavy-ion collisions

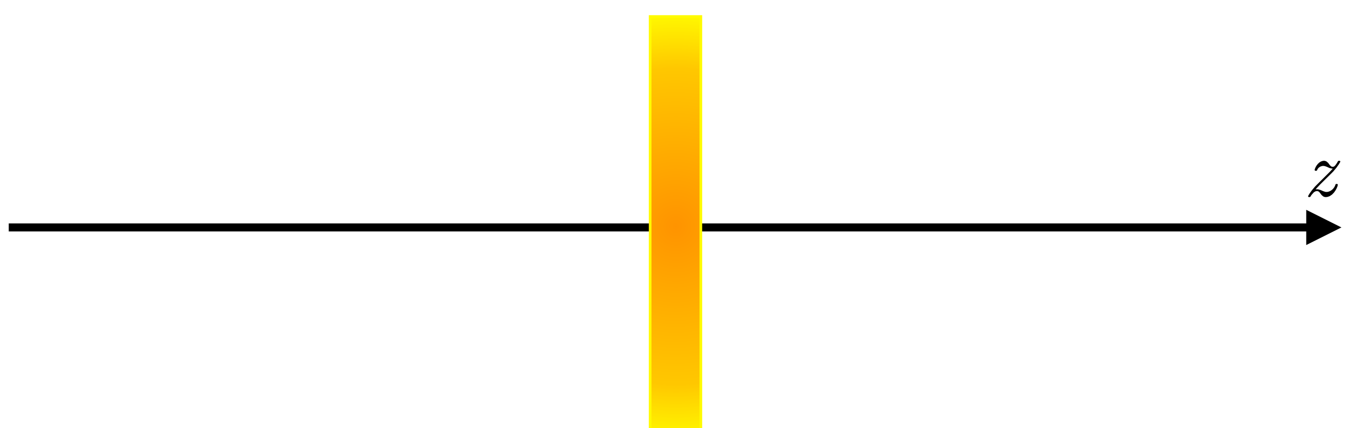
scenarios

- Longitudinal (= along the colliding beam direction) expansion

  ... begins "at once".

Two extreme scenarios: Landau vs. Bjorken.

- In the Landau scenario, the initial condition consists of a slab of matter at rest:



☞ the "stuff" left behind by the colliding nuclei (which have flown away) has been totally stopped.

Fluid dynamics & heavy-ion collisions

scenarios

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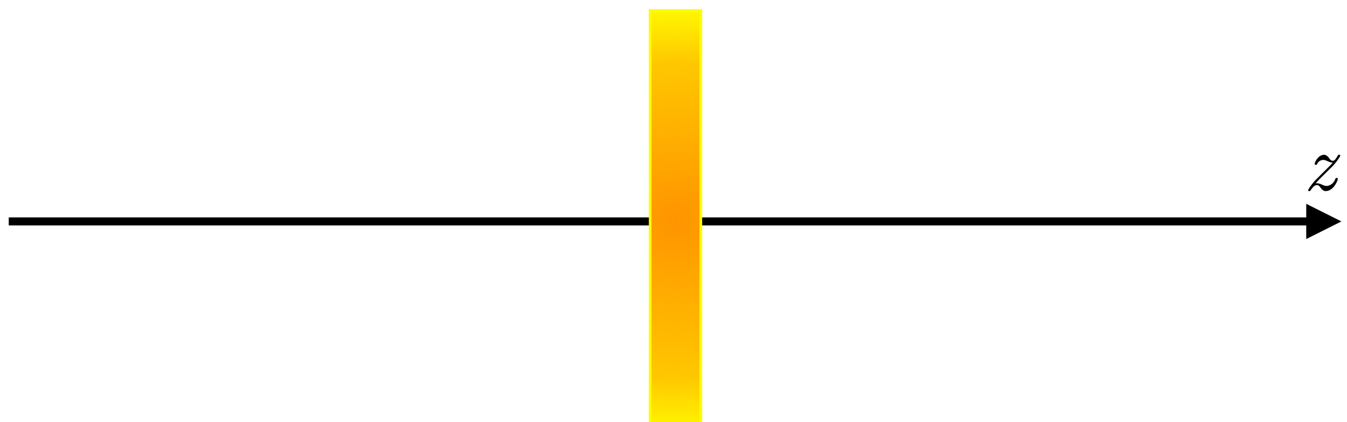


  This slab then starts expanding:

  ☛ first along the z-direction — along which the stronger gradient lies — then also transversely.

More technical details in Wong, arXiv:0809.0517

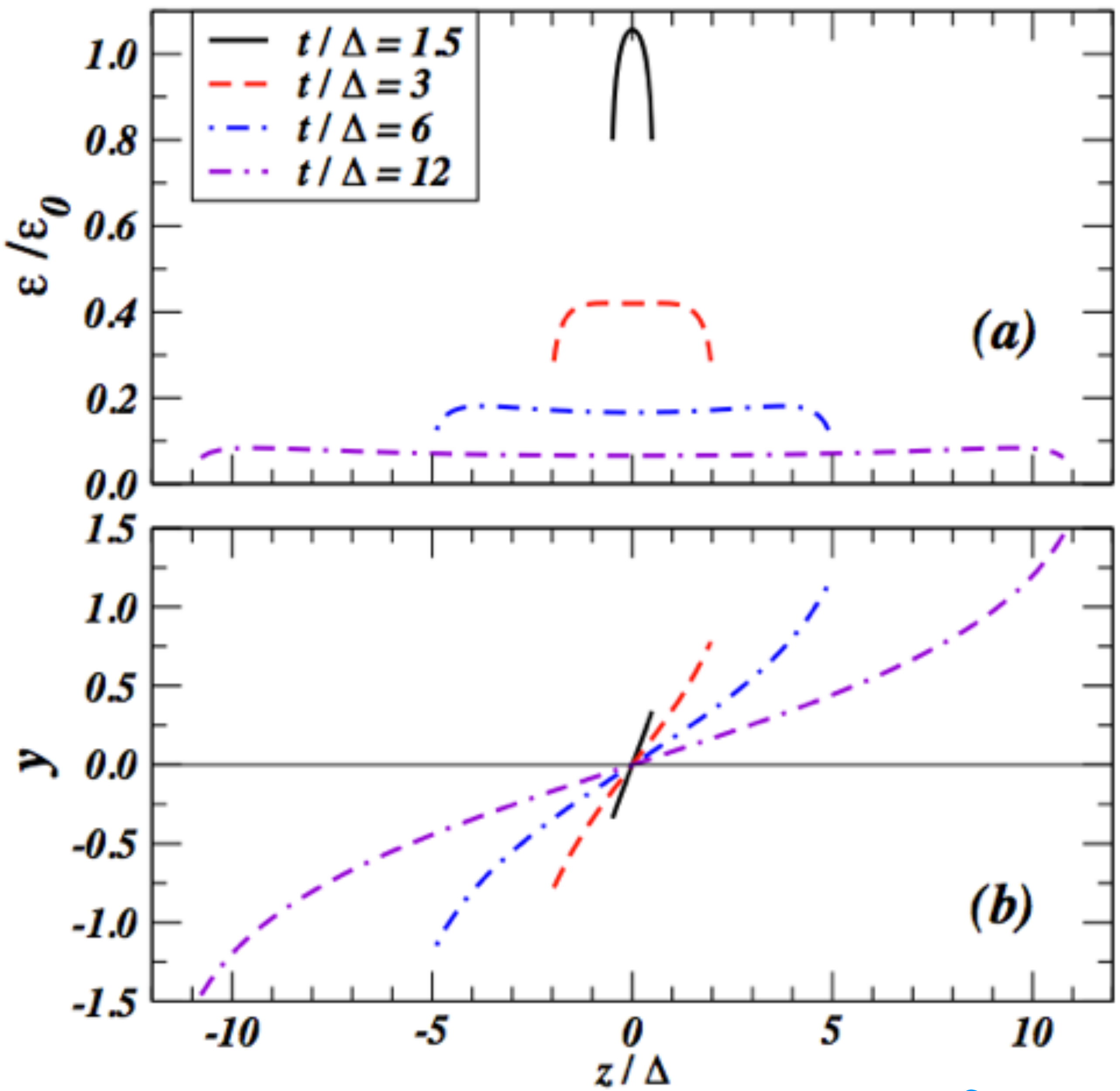
Fluid dynamics & heavy-ion collisions

Landau scenario

Time evolution of longitudinal distributions: (Δ is the initial slab size)

energy density ϵ

"velocity" (in fact, fluid rapidity).



Plot from Wong, arXiv:0809.0517

Fluid dynamics & heavy-ion collisions

scenarios

- Longitudinal (= along the colliding beam direction) expansion

... begins "at once".

Two extreme scenarios: Landau vs. Bjorken.

- In the Bjorken scenario, the initial condition consists of a boost invariant matter distribution in boost invariant motion:

$$\varsigma \equiv \frac{1}{2} \log \frac{t+z}{t-z}$$

(motivated by picture of hadronic strings stretching between the nuclei and decaying uniformly)

The whole problem remains boost invariant as (proper) time goes by.

More technical details in Lecture I!

Fluid dynamics & heavy-ion collisions

scenarios

- Longitudinal (= along the colliding beam direction) expansion

  ... begins "at once".

Two extreme scenarios: Landau vs. Bjorken.

- In the Landau scenario, the fireball expands starting from a state of rest, implying stopping.
- In the Bjorken scenario, boost invariant flow along the longitudinal axis is assumed from the start.

More realistic (= not analytically tractable...) scenarios will lie somewhere in-between, but quite often tend to Bjorken at "large" times.

Note: I am hiding an important issue under the rug

Fluid dynamics & heavy-ion collisions

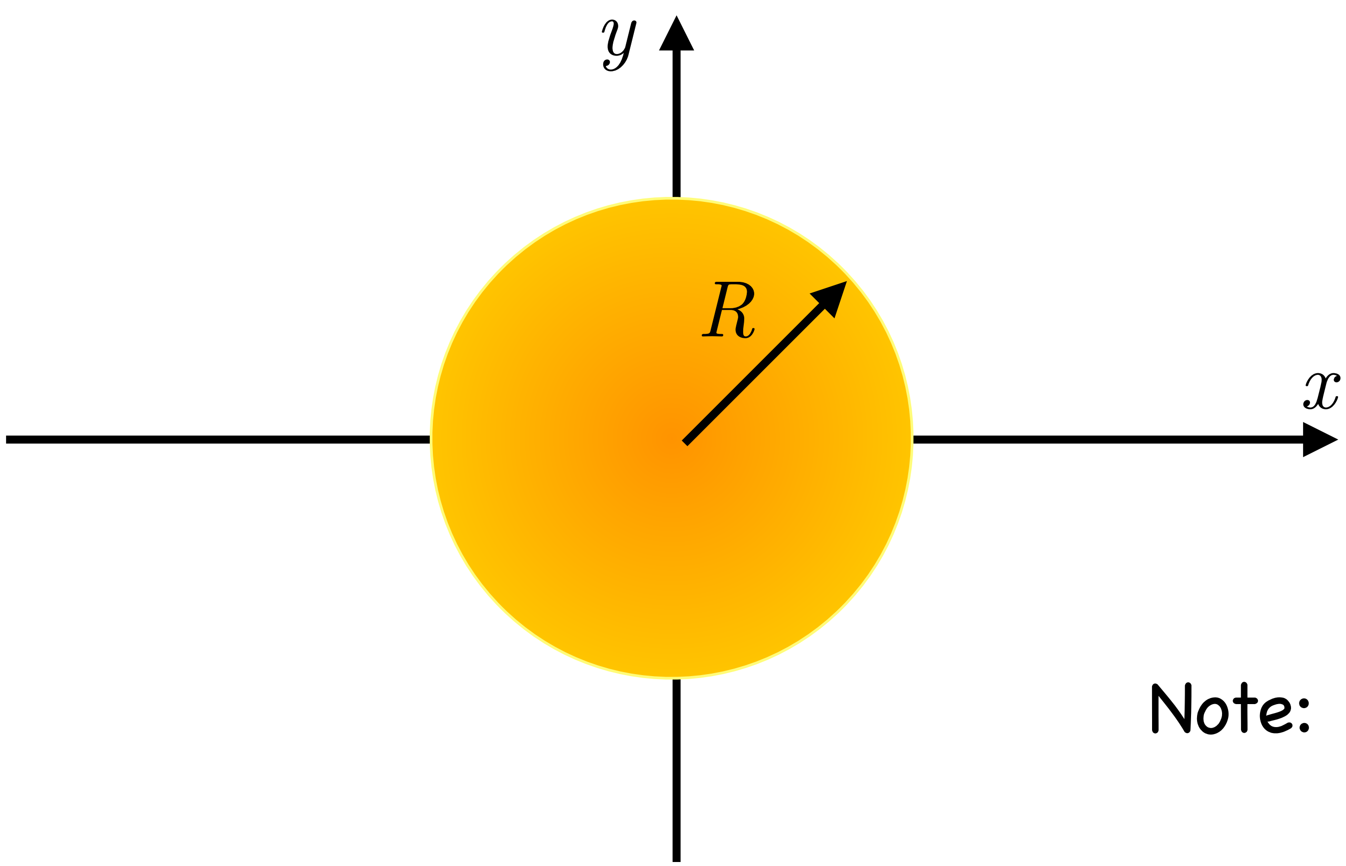
scenarios

- Transverse expansion… begins "later".

or rather, becomes significant "later"

Why?

In the transverse plane, the stuff left behind by the colliding nuclei has a characteristic size: the nuclear radius R



Note: unrealistic picture!
better… later

Fluid dynamics & heavy-ion collisions

scenarios

- Transverse expansion... begins "later".

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Why?

In the transverse plane, the stuff left behind by the colliding nuclei has a characteristic size: the nuclear radius R

☛ The matter at the center "does not know" at once that there is vacuum out there, into which expansion is possible: in fact $R\,/\,c_s$

the propagation of that piece of information takes (at least) $R\,/\,c$.

Less handwaving argument: for matter at the center to start moving, a "rarefaction wave" first has to propagate inwards from the outside.

(interested? check "Riemann problem")

Fluid dynamics & heavy-ion collisions

scenarios

- Transverse expansion... begins "later".

Think of the particles making up the expanding fluid.

(artificial: hydro deals with a continuous medium)

Their average velocity is precisely the flow velocity.

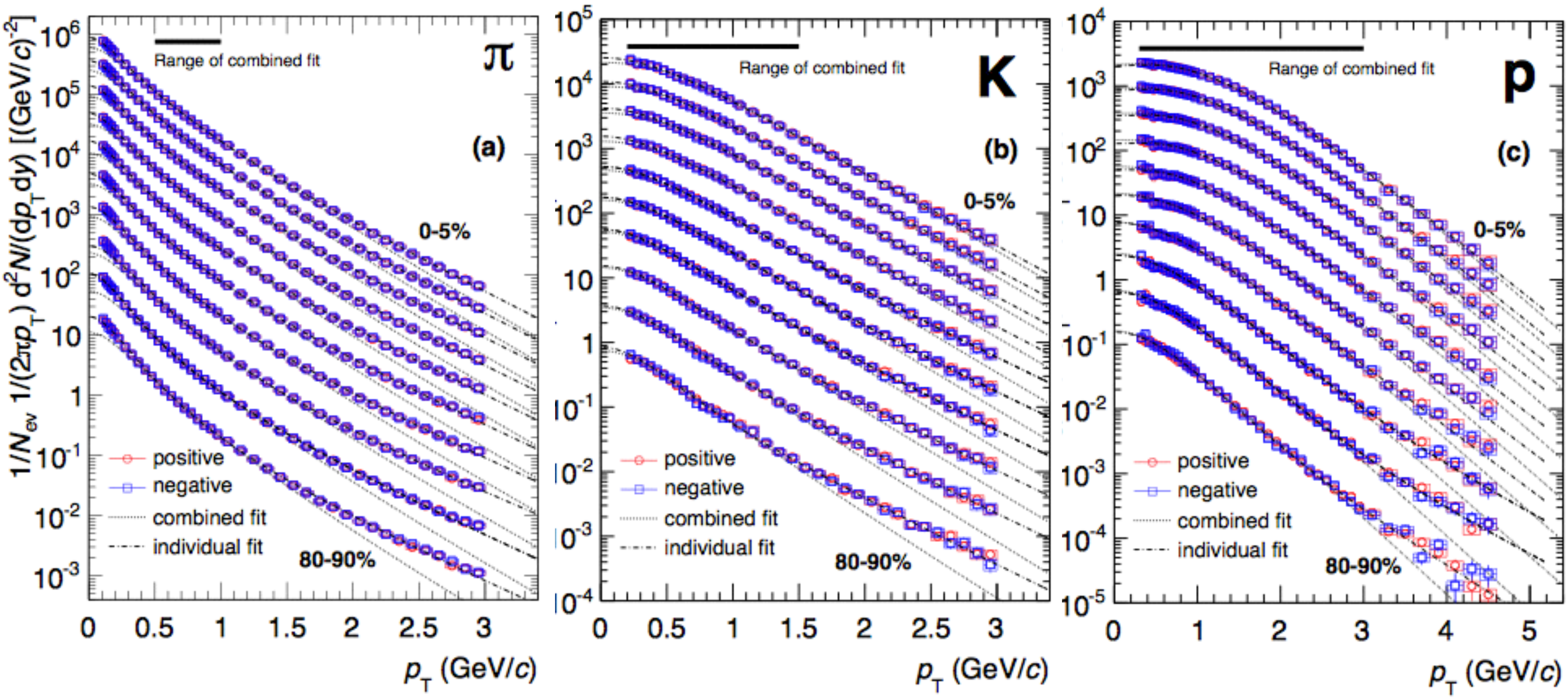
According to the assumption, underlying the use of hydro, of local thermodynamic equilibrium, the (local) momentum distribution of each particle type is "thermal", i.e. given by a Boltzmann factor $\mathrm{e}^{-E/T}$.

☛ in the local rest frame! In the laboratory frame, everything is boosted with the fluid velocity: replace $-E\,/\,T$ by $p^\mu u_\mu(\mathrm{x})\,/\,T$.

☞ leads to transverse spectra $\propto \mathrm{e}^{-(\gamma m_t - \gamma v p_t)/T}$ with $m_t \equiv \sqrt{m^2 + p_t^2}$
i.e. flatter at low momenta for increasing mass.

Transverse momentum spectra

… become increasingly flatter at low momenta with growing particle mass in a hydrodynamical scenario.

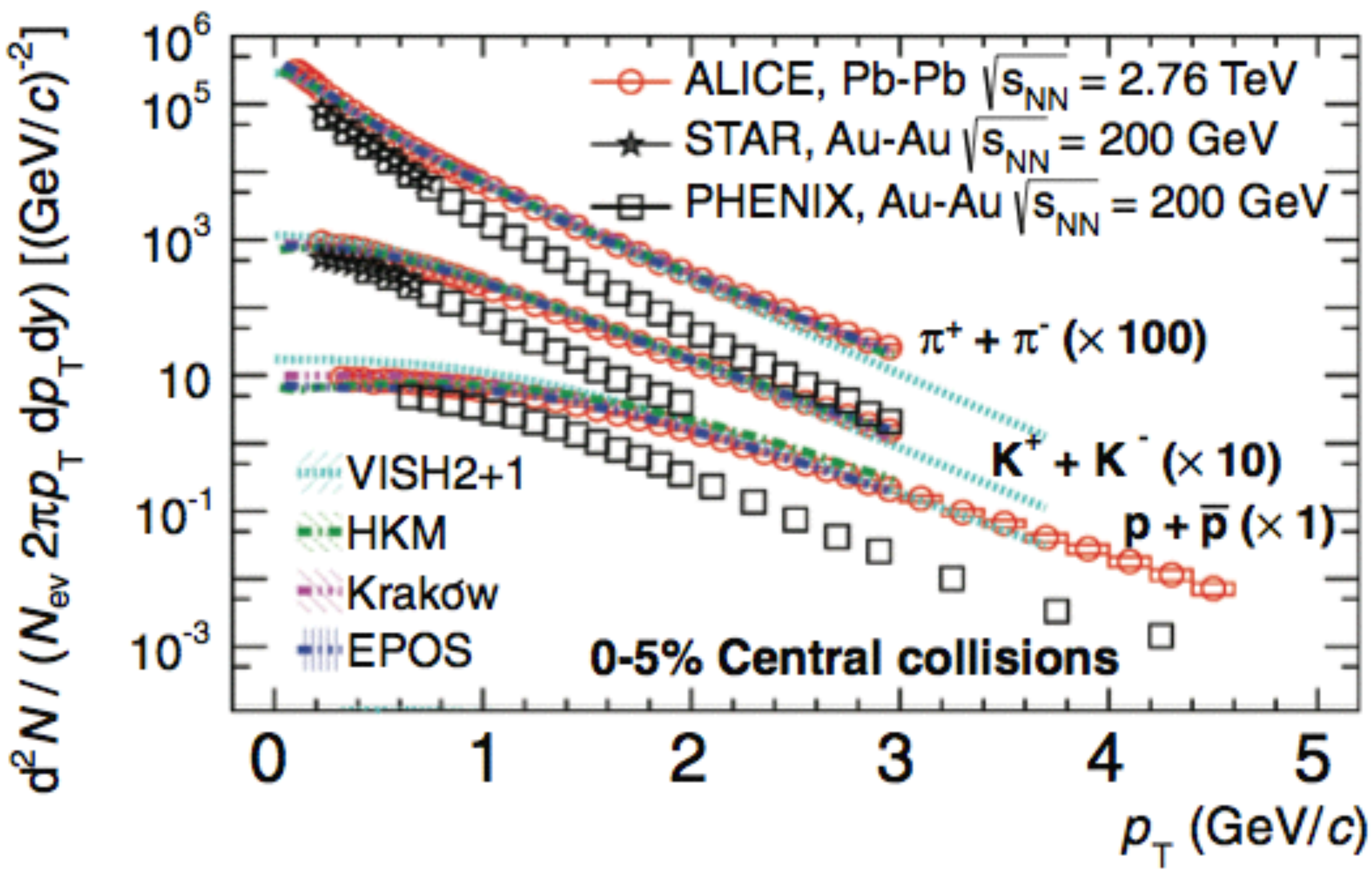


Plot from ALICE Coll., arXiv:1303.0737

Transverse momentum spectra

… become increasingly flatter at low momenta with growing particle mass in a hydrodynamical scenario.

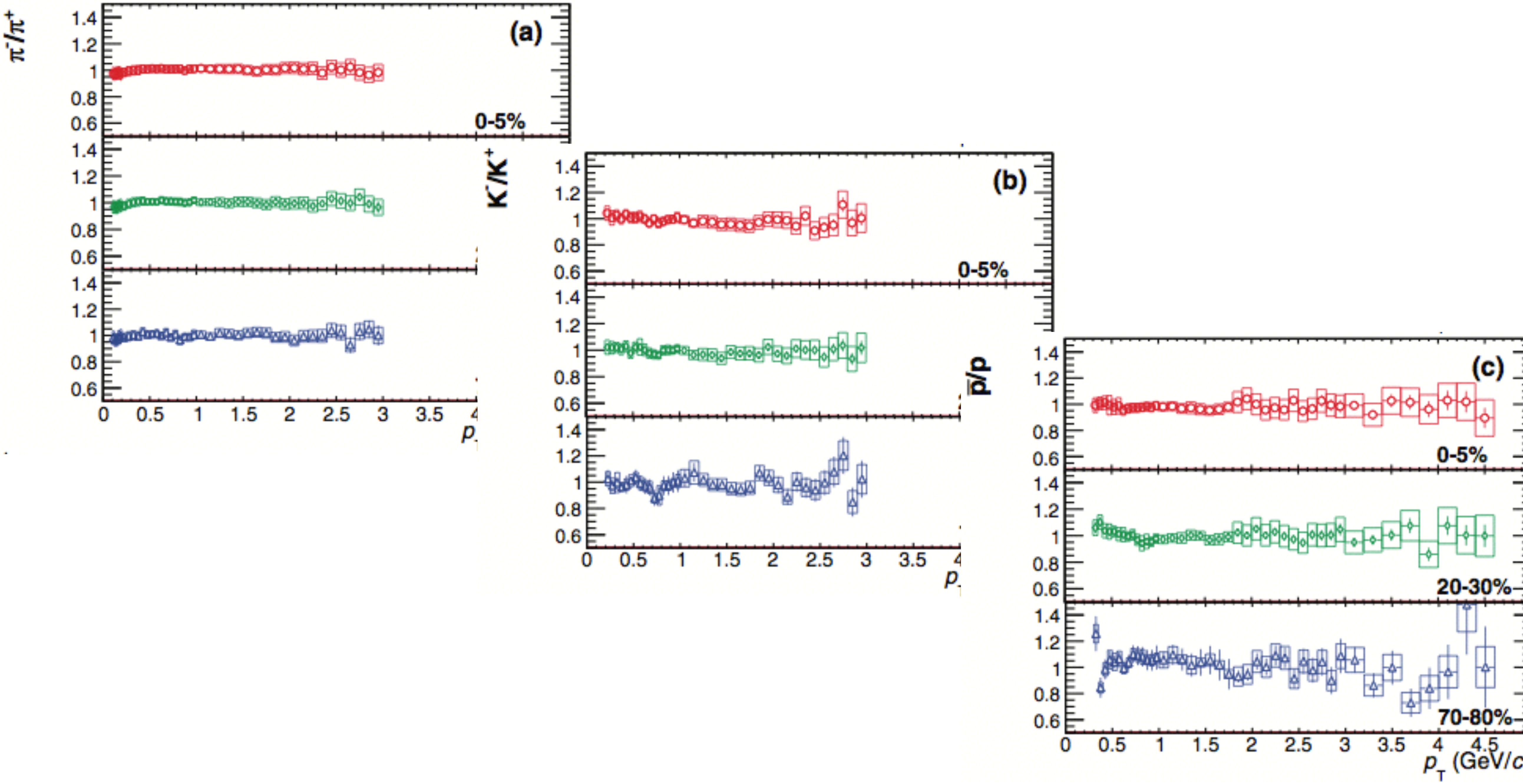
Comparing with real hydro codes:



Plot from ALICE Coll., arXiv:1303.0737

Transverse momentum spectra

… are the same for particles and antiparticles in a hydro scenario.



Plot from ALICE Coll., arXiv:1303.0737

Fluid dynamics & heavy-ion collisions

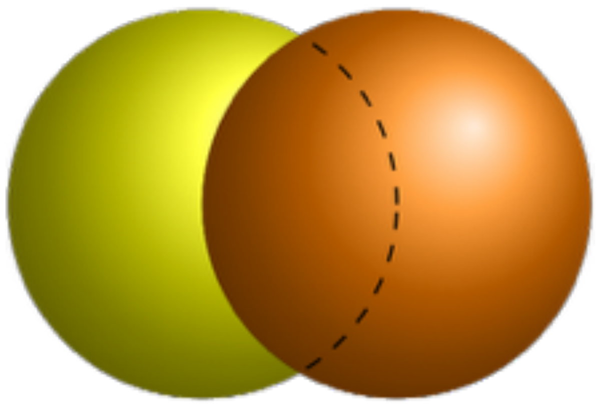
scenarios

- Transverse expansion... begins "later".

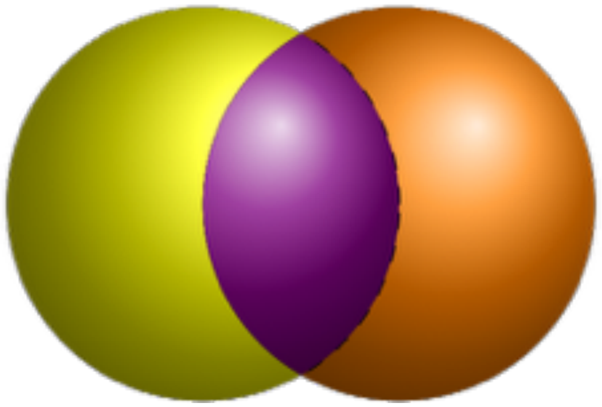
  but may lead to non-trivial phenomena, like "anisotropic flow"

☛ Look at the collision geometry, seen from along the collision axis:

In the generic case, the collision zone (where the nuclei overlap) is asymmetric (in the transverse plane):



$t < 0$



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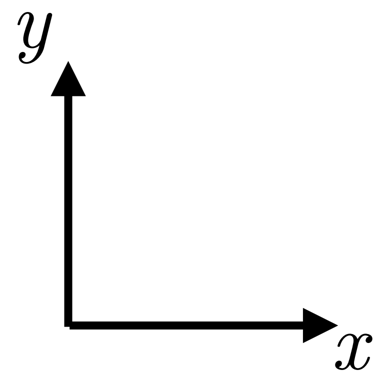

(along the AA impact parameter)

pictures shamelessly stolen from Matt Luzum

Note: these cartoons show the "classical picture" (pre-ca.2010).

Fluid dynamics & heavy-ion collisions

scenarios

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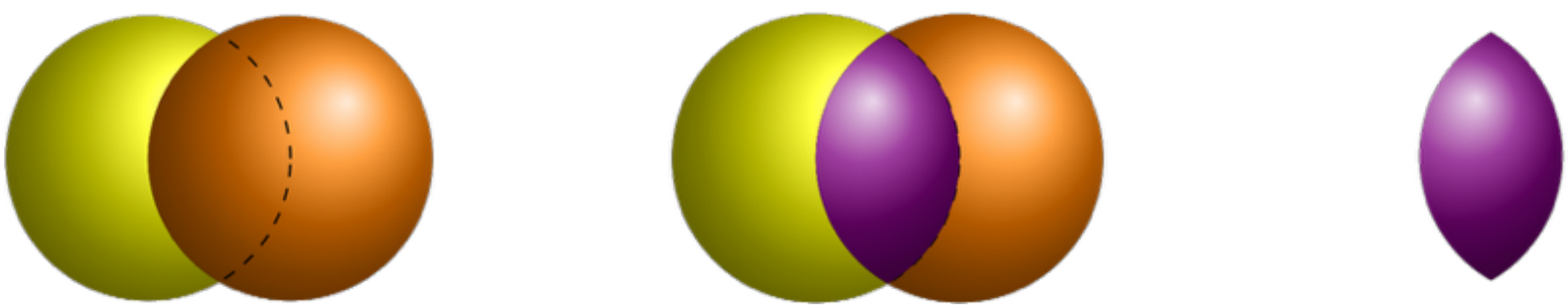

$t < 0$ $t = 0$ $t = 0^+$

The spectator nucleons (= remnants of nuclei) quickly fly away.

Note: these cartoons show the "classical picture" (pre-ca.2010).

pictures shamelessly stolen from Matt Luzum

Fluid dynamics & heavy-ion collisions

anisotropic flow

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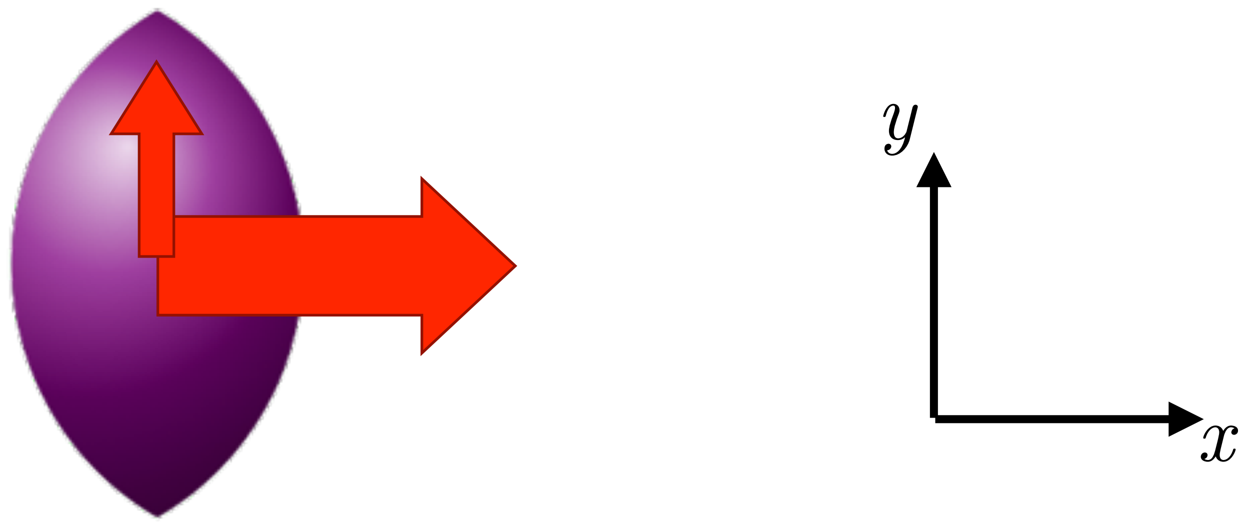

There is matter in the middle, with a given pressure, and vacuum outside: larger pressure gradient along the x-direction than along y.

Invoke the Euler eq.: $\big[\epsilon(\mathsf{x}) + \mathcal{P}(\mathsf{x})\big] u^\mu(\mathsf{x}) \partial_\mu u^\nu(\mathsf{x}) + \nabla^\nu \mathcal{P}(\mathsf{x}) = 0$

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Fluid dynamics & heavy-ion collisions

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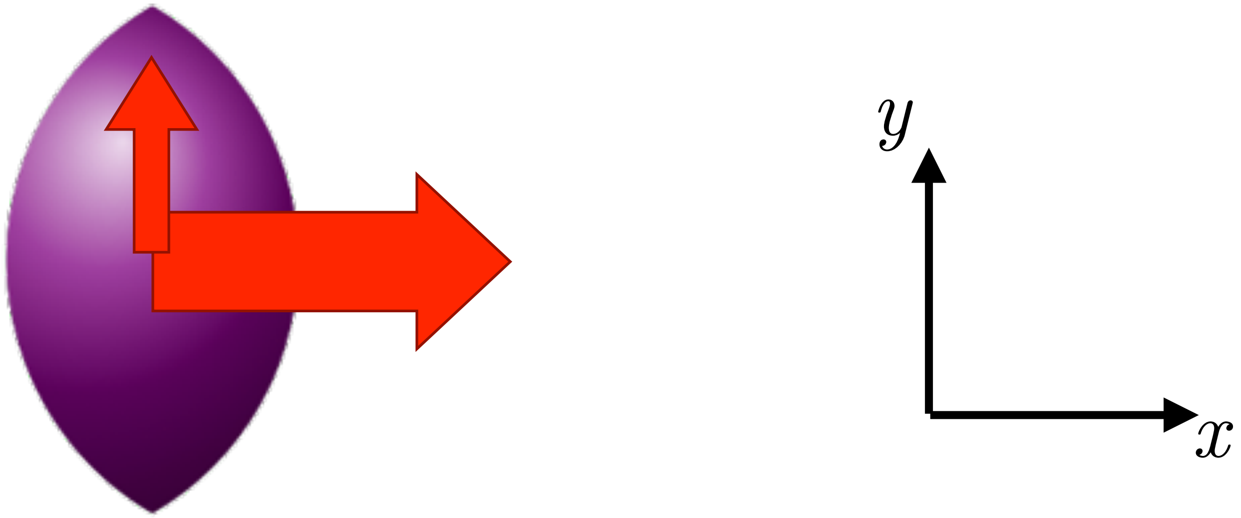


There is matter in the middle, with a given pressure, and vacuum outside: larger pressure gradient along the x-direction than along y.

Invoke the Euler eq.: fluid acceleration $\propto$ pressure gradient

Note: these cartoons show the "classical picture" (pre-ca.2010).

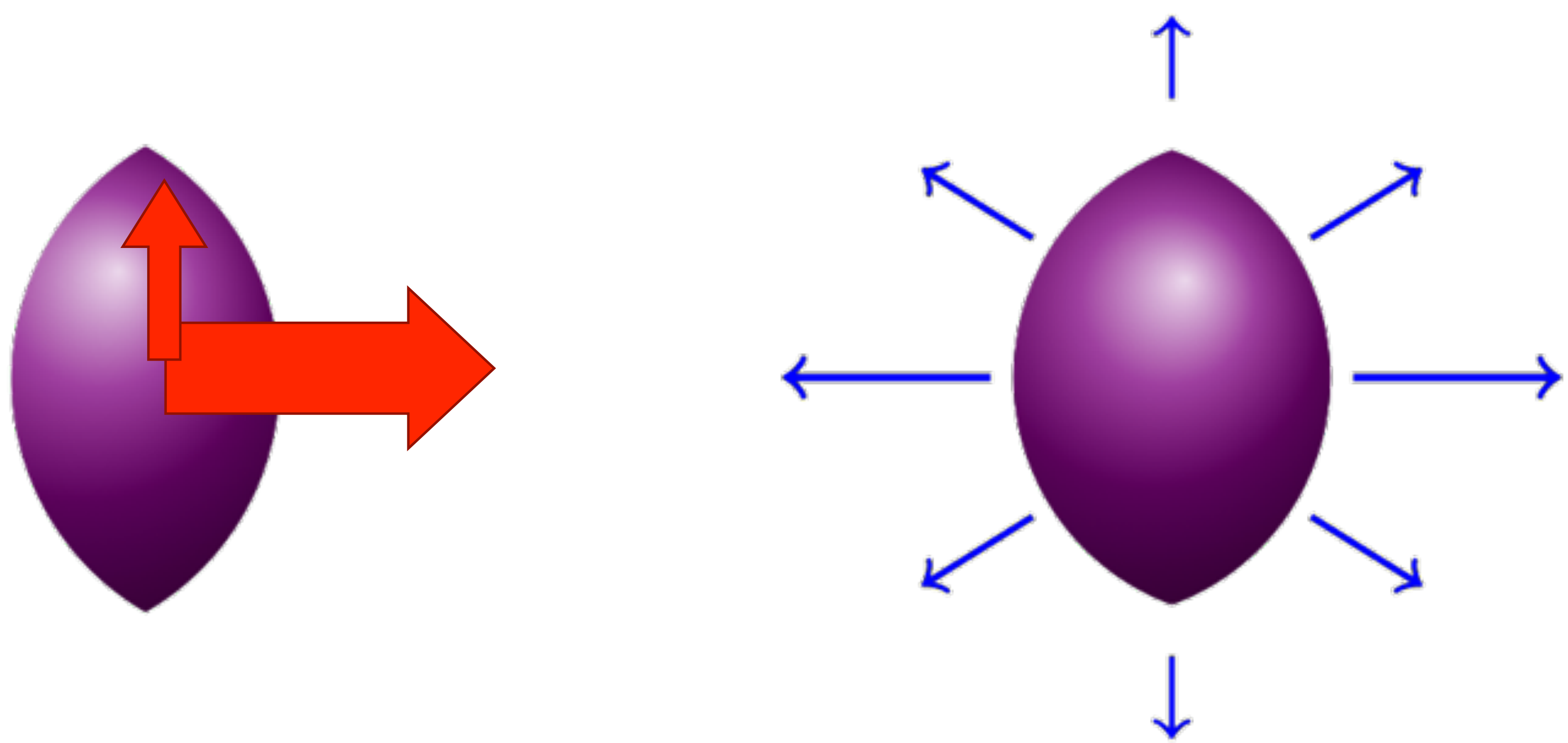
pictures shamelessly stolen from Matt Luzum

Fluid dynamics & heavy-ion collisions

anisotropic flow

In the generic case, the collision zone (where the nuclei overlap) is asymmetric (in the transverse plane):

$\Rightarrow$ anisotropic transverse expansion



$t = 0^+$ $\qquad$ $t > 0$

pictures shamelessly stolen from Matt Luzum

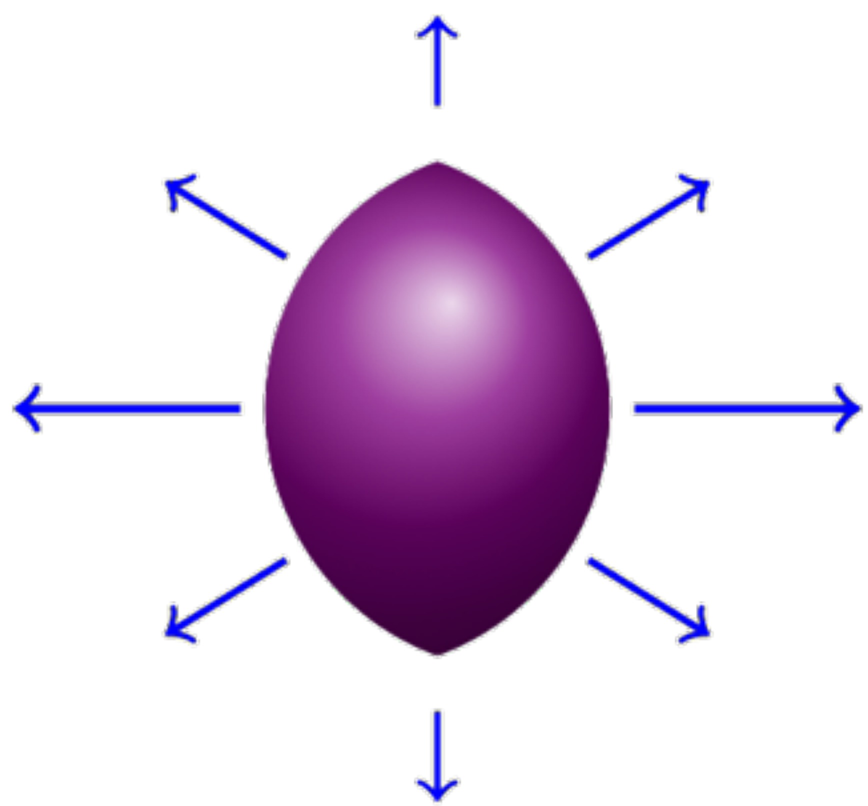
Note: these cartoons show the "classical picture" (pre-ca.2010).

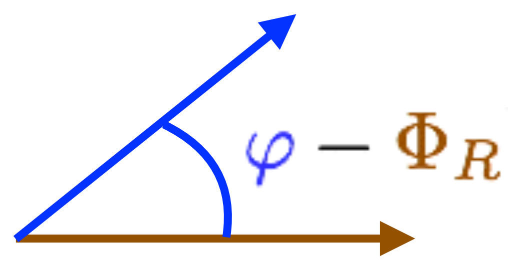
Fluid dynamics & heavy-ion collisions

anisotropic flow

In the generic case, the collision zone (where the nuclei overlap) is asymmetric (in the transverse plane):

$\Rightarrow$ anisotropic energy / momentum flux: more along $\varphi - \Phi_R = 0$ or 180° than along $\varphi - \Phi_R = \pm 90°$.





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Fluid dynamics & heavy-ion collisions

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$\Rightarrow$ anisotropic energy / momentum flux: more along $\varphi - \Phi_R = 0$ or 180° than along $\varphi - \Phi_R = \pm 90°$.

quantified with coefficient v_2 (more precise definition later).

because of $\frac{2\pi}{2}$-periodicity

pictures shamelessly stolen from Matt Luzum

Fluid dynamics & heavy-ion collisions

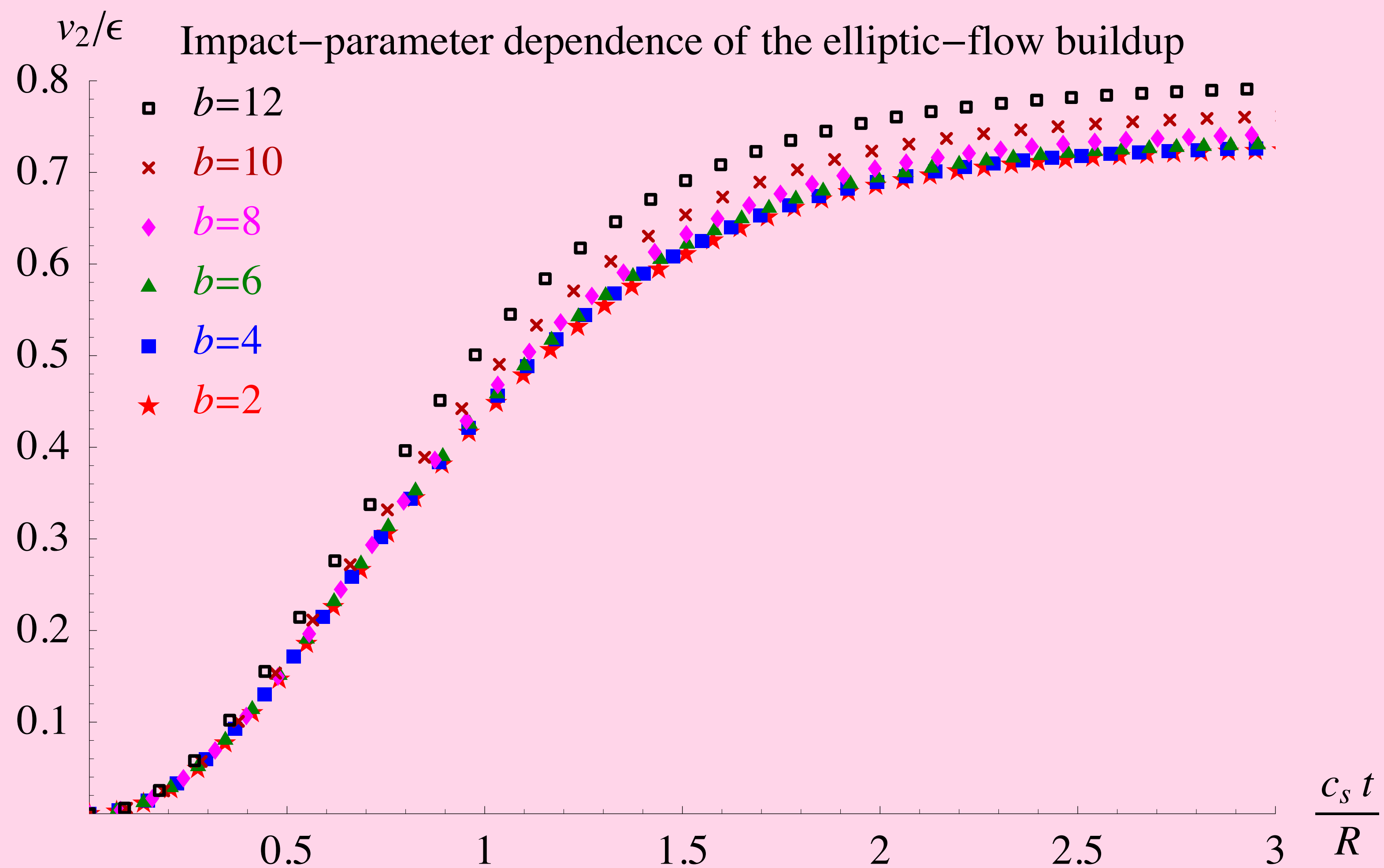
anisotropic flow

In the gene… …ei overlap) is
asymmetric (…

⇒ anisotropi… … = 0 or 180°
than along φ…

quantified wi… …r).

irrelevant side-remark:



to show that the typical time scale is $R\,/\,c_s$

pictures shamelessly stolen from Matt Luzum

Fluid dynamics & heavy-ion collisions

anisotropic flow

In the generic case, the collision zone (where the nuclei overlap) is asymmetric (in the transverse plane):

$\Rightarrow$ anisotropic energy / momentum flux, quantified with coefficient v_2, which should increase with the nucleus-nucleus impact parameter:

Now, the impact parameter is correlated with the particle multiplicity.

Fluid dynamics & heavy-ion collisions

anisotropic flow

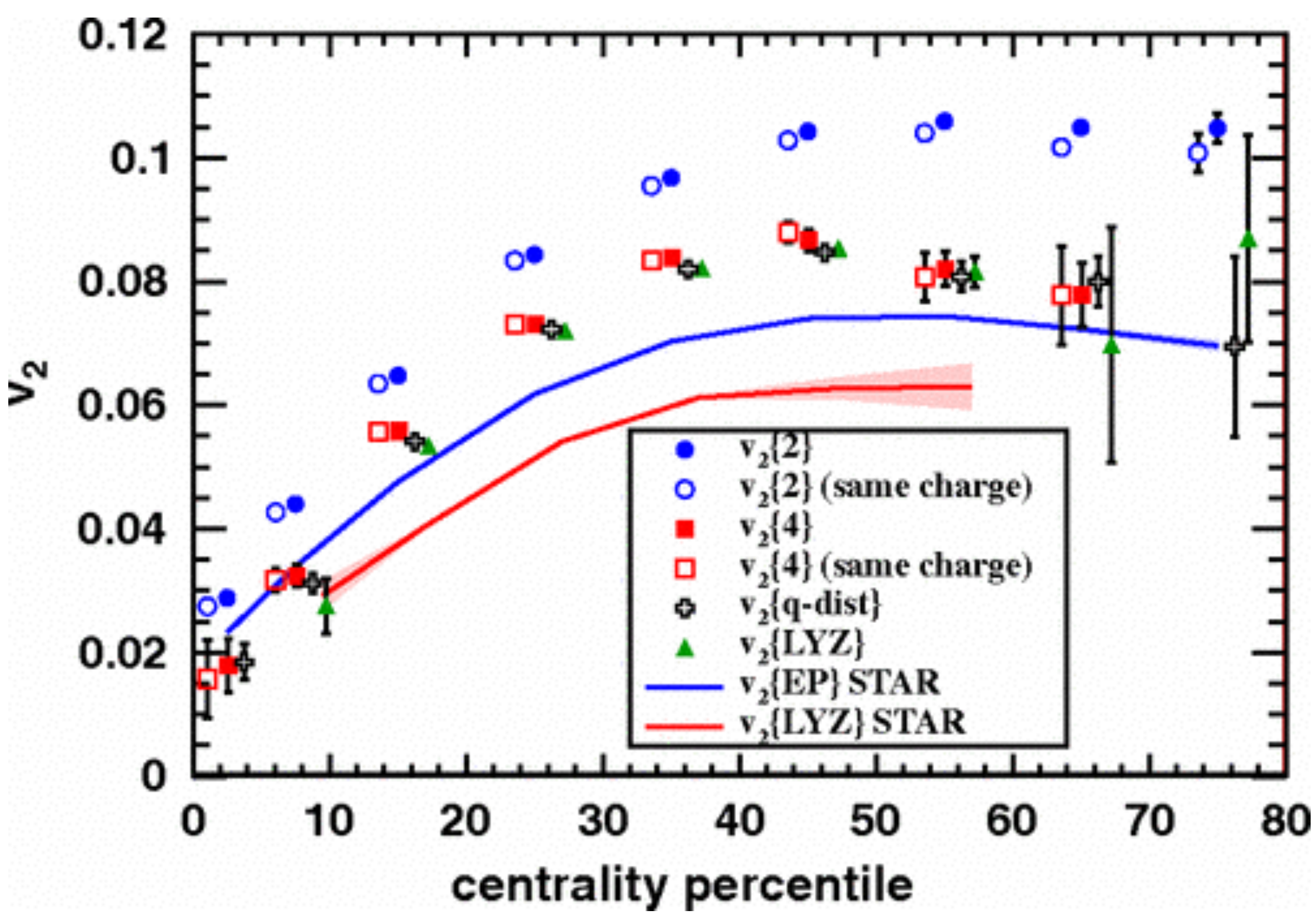
In the generic case, the collision zone (where the nuclei overlap) is asymmetric (in the transverse plane):

$\Rightarrow$ anisotropic energy / momentum flux, quantified with coefficient v_2, which should increase with centrality

Fluid dynamics & heavy-ion collisions

anisotropic flow

⇒ anisotropic energy / momentum flux, quantified with coefficient v_2, which should increase with centrality



Plot from ALICE Coll., Phys. Rev. Lett. 105 (2010) 252302

Elements of fluid dynamics for the modeling of heavy-ion collisions

Lecture II:

- Overview on the dynamics of relativistic perfect fluids
- Application of perfect relativistic fluid dynamics to the description of high-energy nucleus-nucleus collisions

phenomenology & experiment

  - modeling of the bulk of the matter created in the collisions
  - yields a surprisingly good rendering of some measurements
  - ... at the cost of introducing elements that are not within perfect hydrodynamics

- Beyond perfect relativistic fluid dynamics

Fluid dynamics & heavy-ion collisions

hidden "details"

Fluid dynamical simulations yield a good description of experimental results, yet I have till now omitted a few important issues...

- Fluid dynamics describes the evolution of a continuous medium, yet experiments measure (energy, momentum, tracks...) of particles.

  ☛ how can / should one relate both?

- Fluid dynamics describes the evolution of a continuous medium, starting from an initial condition.

  ☛ which initial condition?

- Fluid dynamics describes the evolution of a continuous medium at thermodynamic equilibrium.

  ☛ how do we reach a thermalized "initial state"?

Fluid dynamics & heavy-ion collisions

hidden "details"

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  ☞ BEFORE the hydro stage

- Fluid dynamics describes the evolution of a continuous medium, yet experiments measure (energy, momentum, tracks...) of particles.

  ☛ how can / should one relate both?

  ☞ END of the hydro stage

Space-time evolution of an ultrarelativistic heavy-ion collision

What happens before and after the hydro stage?

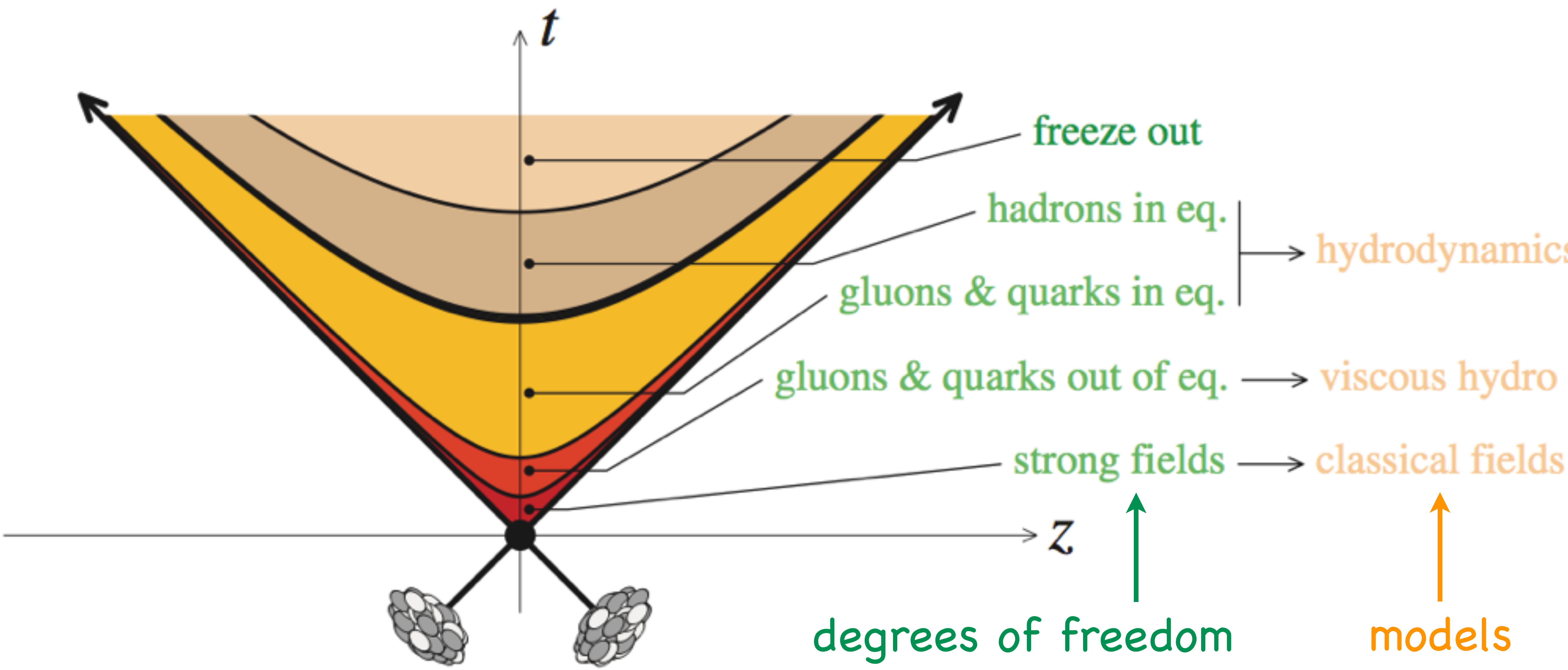

Figure taken from F.Gelis, [Int. J. Mod. Phys. A **28** (2013) 133001]

Fluid dynamics & heavy-ion collisions

hidden "details"

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  ☛ how can / should one relate them?

  ☞ END of the hydro stage

Crucial, complicated, unsolved issue!

in Lecture III: phenomenological idea

to extend fluid dynamics to earlier times

Fluid dynamics & heavy-ion collisions

hidden "details"

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Fluid dynamics & heavy-ion collisions

initial conditions of hydro

- Fluid dynamics describes the evolution of a continuous medium, starting from an initial condition.

  ☛ which initial condition?

  There are several models / pictures on the market, some trying to tackle simultaneously the thermalization / hydrodynamization* issue, most disconnected thereof and merely parameterizing the "initial state".

  ☞ Turn the reasoning round, and argue that you can constrain the initial state from the experimental data!

* nice(?) creation of the heavy-ion-collisions field: reaching a state where fluid dynamics is applicable, although the medium may still be non-thermalized.

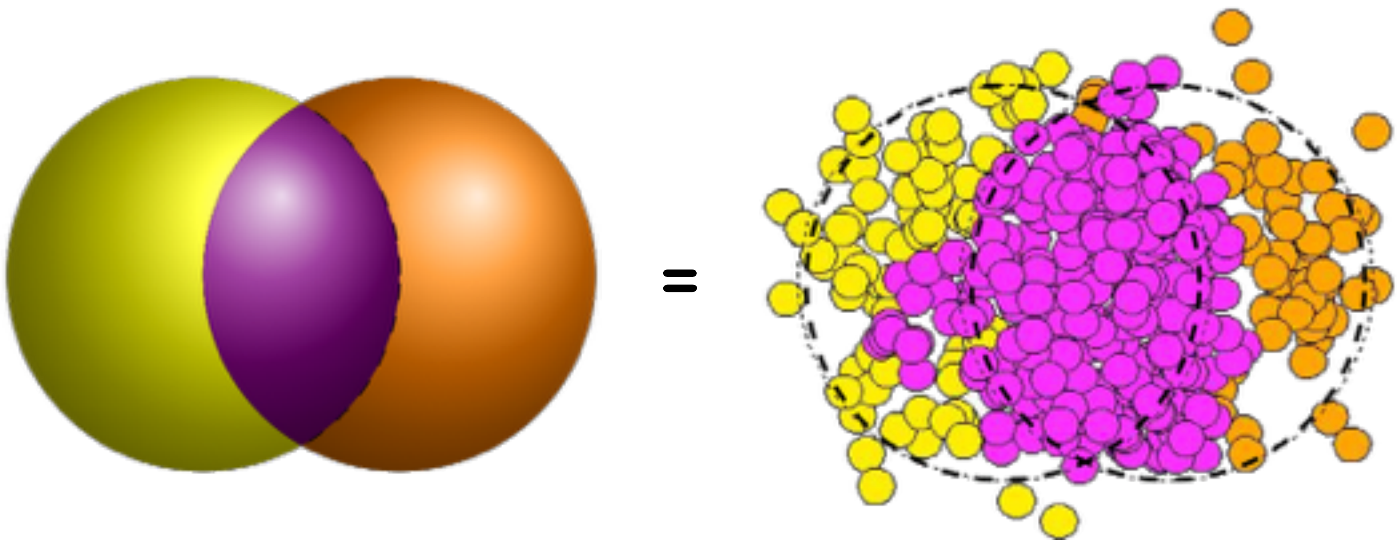
Fluid dynamics & heavy-ion collisions

"Glauber picture" for initial conditions

The colliding nuclei are not continuous matter distributions, they are collections of (bound) nucleons in constant motion.

The collision of the nuclei amounts to many nucleon-nucleon collisions.*
At ultrarelativistic energies, the nucleus-nucleus collision is so quick ($\sim 2R/\gamma$) that the nucleon positions are frozen:

collision $\Leftrightarrow$ snapshot of the position



pictures shamelessly stolen from Matt Luzum

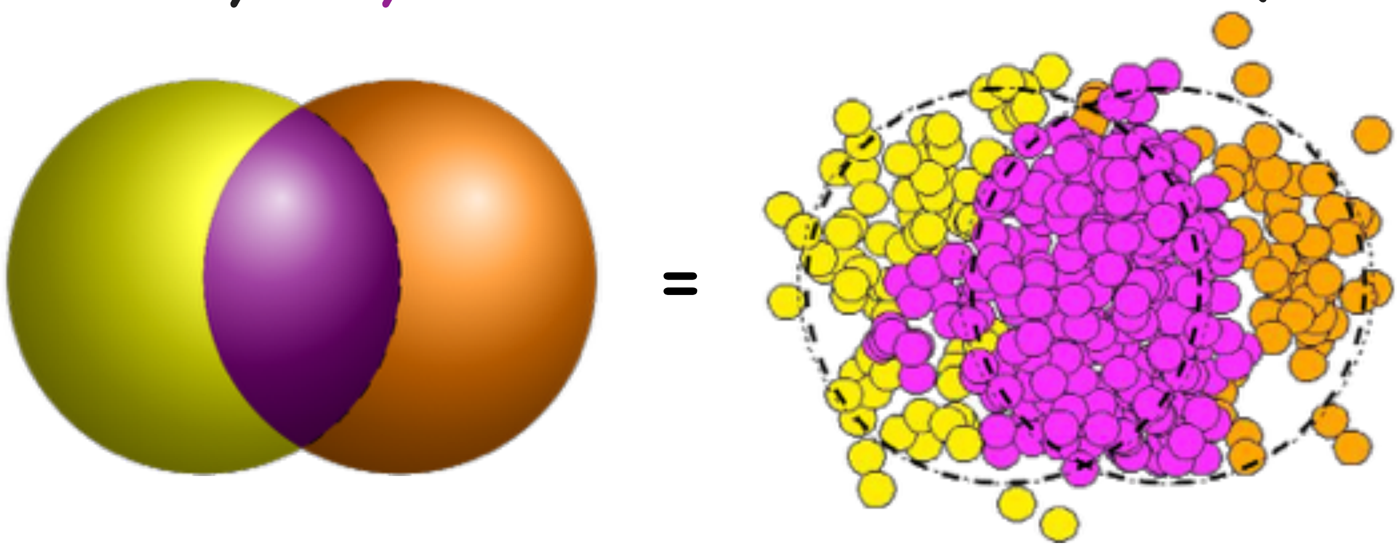
* 1. … as a first approximation: a nucleon that has already interacted once may no longer be in the same state
2. Should we count quarks instead of nucleons?

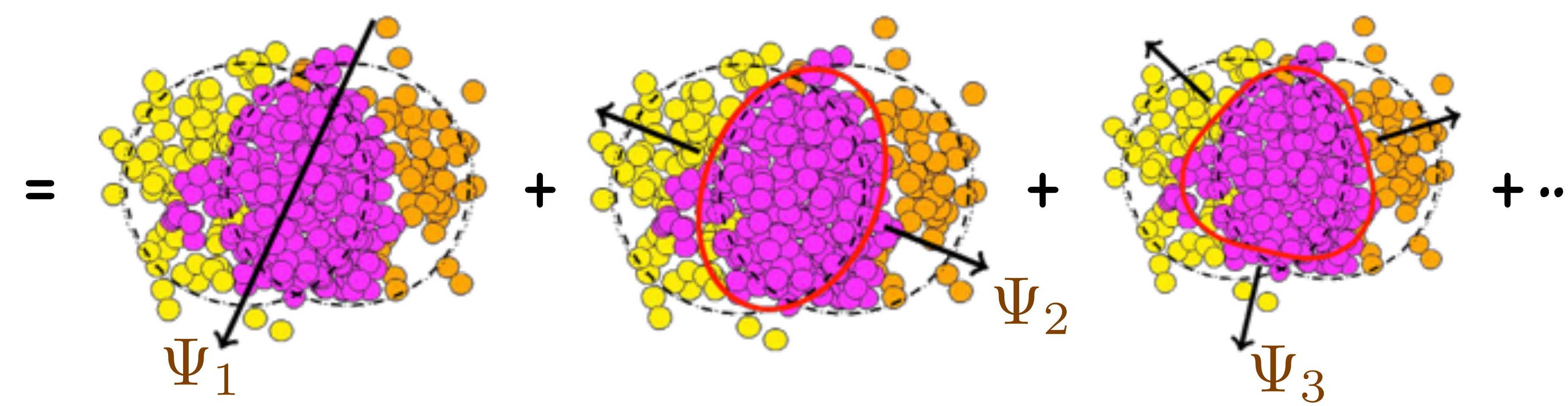
Fluid dynamics & heavy-ion collisions

"Glauber picture" for initial conditions & anisotropic flow

Newer picture (since 2010) in collisions of "large" systems:

Initially asymmetric collision zone (in the transverse plane)





$\Rightarrow$ leads to anisotropic expansion with 180°, 120°... symmetry.

quantified with coefficients v_2, v_3, v_4... (v_1 as well, left aside hereafter)

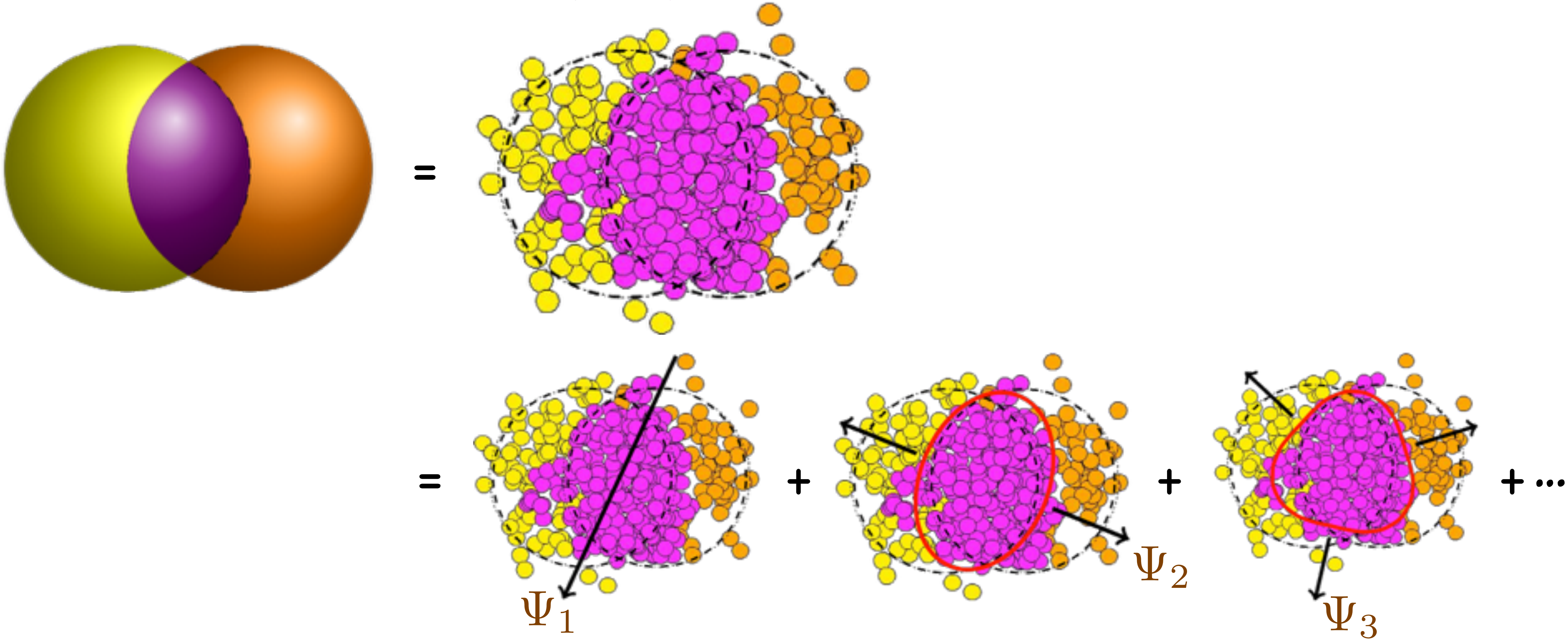
pictures shamelessly stolen from Matt Luzum

Fluid dynamics & heavy-ion collisions

"Glauber picture" for initial conditions & anisotropic flow

Newer picture (since 2010) in collisions of "large" systems:

Fluctuations in the initially asymmetric transverse collision zone



⇒ lead to "irregular" anisotropic expansion quantified with coefficients v_2, v_3, v_4... (v_1 as well, left aside hereafter)

Fluid dynamics & heavy-ion collisions

"Glauber picture" for initial conditions & anisotropic flow

Newer picture (since 2010) in collisions of "large" systems:

Fluctuations in the initially asymmetric transverse collision zone

⇒ lead to "irregular" anisotropic expansion quantified with coefficients v_2, v_3, v_4... (v_1 as well, left aside hereafter).

☞ Turning the reasoning round, by measuring the flow coefficients v_2, v_3, v_4..., you can constrain the initial state.

Note: the flow anisotropies do not evolve similarly in the hydrodynamic expansion. "Higher order" coefficients parameterize increasingly fine structures, which are more affected (= dampened) by viscosity.

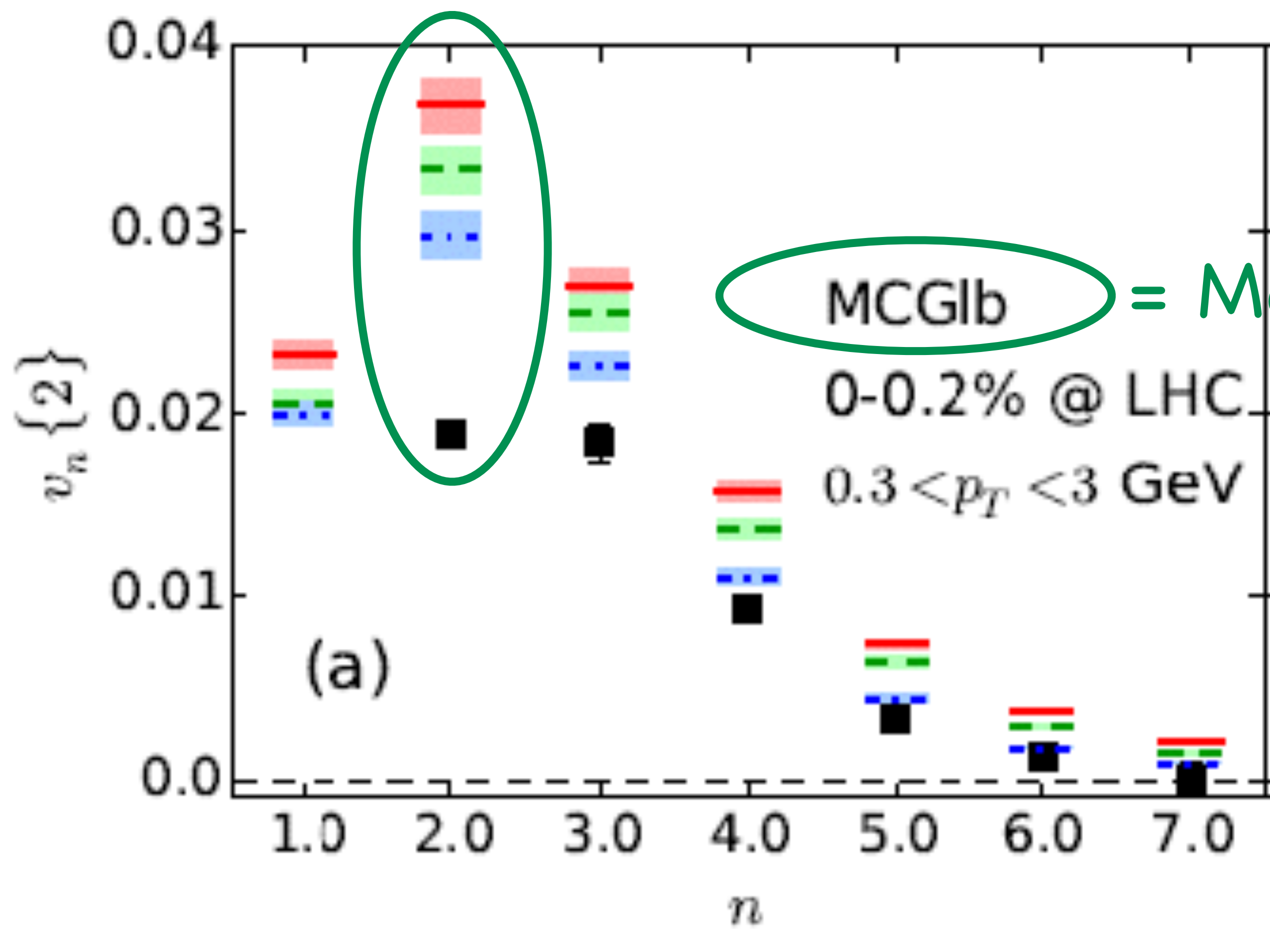
☞ twofold game: constrain initial state & medium properties!

Fluid dynamics & heavy-ion collisions

"Glauber picture" for initial conditions & anisotropic flow

☞ Turning the reasoning round, by measuring the flow coefficients v_2, v_3, v_4..., you can constrain the initial state.

"tension"



= Monte Carlo Glauber

Fluid dynamics & heavy-ion collisions

hidden "details"

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  ☛ how can / should one relate both?

  ☞ END of the hydro stage

Fluid dynamics & heavy-ion collisions

conversion from a fluid to a collection of particles

Idea: "at some point", each small local fluid cell breaks up into a set of particles, which afterwards are treated as such:

freeze-out / decoupling

extreme description: after freeze-out, the particles are free

Note: this is **not** a real physical transition (like e.g. a phase transition), it is a change in our choice of model!

Simplest scenario:

- Freeze-out happens when the fluid crosses a given hypersurface Σ in its space-time evolution.
- At each point on Σ, the corresponding (infinitesimal) fluid volume emits particles according to a phase-space distribution $f(x,p)$.

Fluid dynamics & heavy-ion collisions

conversion from a fluid to a collection of particles

Simplest scenario:

- Freeze-out happens when the fluid crosses a given hypersurface Σ in its space-time evolution

  ☛ how is this hypersurface chosen?

- At each point on Σ, the corresponding (infinitesimal) fluid volume emits particles according to a phase-space distribution f(x,p).

  ☛ which distribution?

☞ Cooper–Frye prescription for the momentum distribution of emitted particles

$$E_{\vec{p}}\frac{\mathrm{d}^3N}{\mathrm{d}^3\vec{p}} = \frac{g}{(2\pi)^3}\int_\Sigma f\left(\frac{\mathsf{p}\cdot\mathsf{u}(\mathsf{x})}{T}\right)\mathsf{p}\cdot\mathrm{d}^3\sigma(\mathsf{x})$$

Fluid dynamics & heavy-ion collisions

conversion from a fluid to a collection of particles

Simplest scenario:

- Freeze-out happens when the fluid crosses a given hypersurface Σ
i

- ne

irrelevant side-remark:

the anisotropic flow coefficients v_n are conventionally defined by

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T\mathrm{d}p_T\mathrm{d}y}\left[1+2v_1\cos(\varphi-\Phi_R)+2v_2\cos 2(\varphi-\Phi_R)+\cdots\right]$$

☞ Cooper–Frye prescription for the momentum distribution of emitted particles

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Fluid dynamics & heavy-ion collisions

conversion from a fluid to a collection of particles

Simplest scenario:

- Freeze-out happens when the fluid crosses a given hypersurface Σ in its space-time evolution

☛ how is this hypersurface chosen?

Physical choice: the validity of the fluid description is governed by the Knudsen number, so that freeze-out should take place when Kn is no longer $\ll 1$.

Hard to implement in practice!

☞ usual choices: constant temperature / constant energy or entropy density...

☛ become parameter(s) of the modeling!

Fluid dynamics & heavy-ion collisions

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☛ which phase-space distribution?

(clearly affects the particle distribution!)

Obvious choice: equilibrium distributions (Bose–Einstein, Fermi–Dirac): each cell is at thermodynamic equilibrium.

Not innocent! Deserves further discussion in Lecture III

Fluid dynamics & heavy-ion collisions

- The large number of particles created in a nucleus-nucleus collision at the LHC justifies the use of a continuous medium description, and thereby of fluid dynamics, for the bulk of particles — at least as a first approach.
- Data tells us that there are genuinely collective dynamical effects at play: anisotropic flow...

  ☛ use of hydro not fully absurd from the start
- The equations of fluid dynamics have to be fed with information:
  - equation of state (under control)
  - transport coefficients and related
  - initial conditions
- Eventually, the fluid model has to be matched (smoothly!) to a particle description.