High-energy heavy-ion collisions.
Selected phenomenological aspects

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- Lecture I. Introduction - First steps
- Lecture II. "Collective flow"
  - Anisotropic particle-emission pattern
  - A genuinely collective effect:
    characterization with the help of "Lee-Yang zeroes"
  - What can we learn from the magnitude of anisotropic flow?
- Lecture III. "Hard probes": high-$p_{\rm T}$ particles

General remarks

- Throughout this lecture, I shall consider collisions between identical (A-A), spherical nuclei:

^{197}Au-^{197}Au, ^{208}Pb-^{208}Pb... but not ^{238}U-^{238}U (deformed nucleus)

- For simplicity, I shall forget about the longitudinal coordinate along the beam (rapidity), and only focus on transverse aspects.

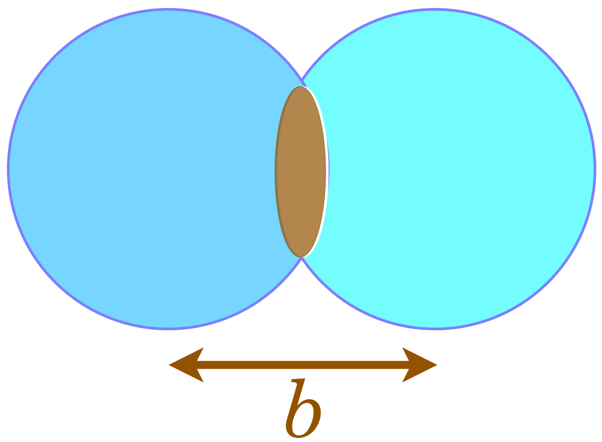
☞ transverse momentum $\mathbf{p}_T$

(more drastically, I shall most often sit at mid-rapidity)

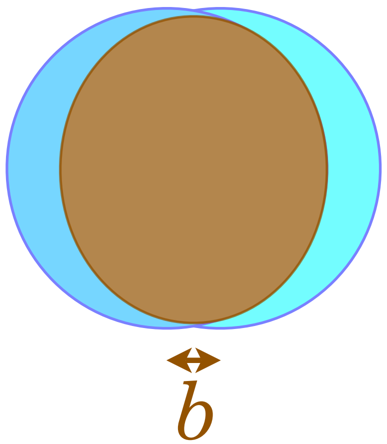
Heavy-ion collisions: geometry

Heavy nuclei have a finite radius!

☞ In a collision the impact parameter plays a role:



- the nuclei might barely graze each other (large impact parameter, "peripheral" collision)



- or the collision might be almost head-on (small impact parameter, "central" collision)

The (almond-shaped) overlap regions of the nuclei are different in either case (size, eccentricity...).

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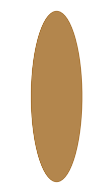

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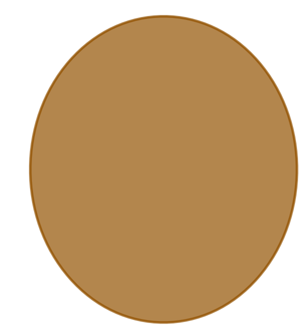

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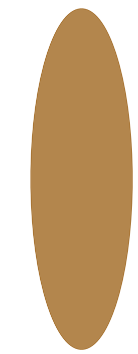

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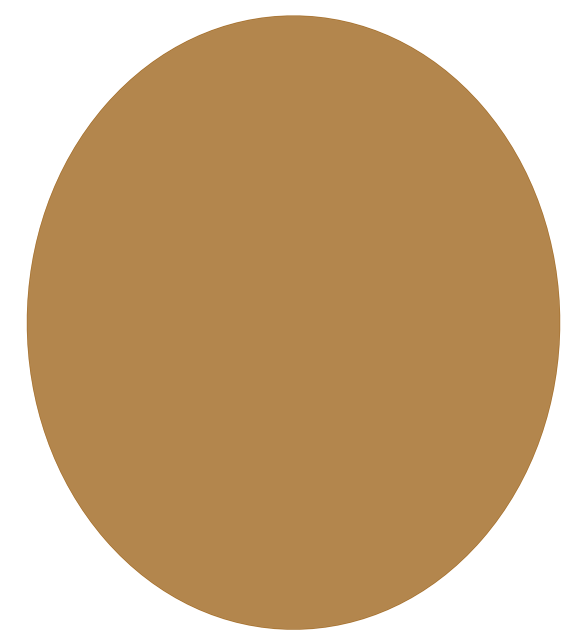

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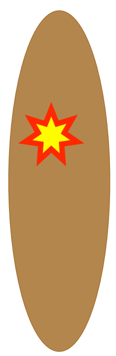

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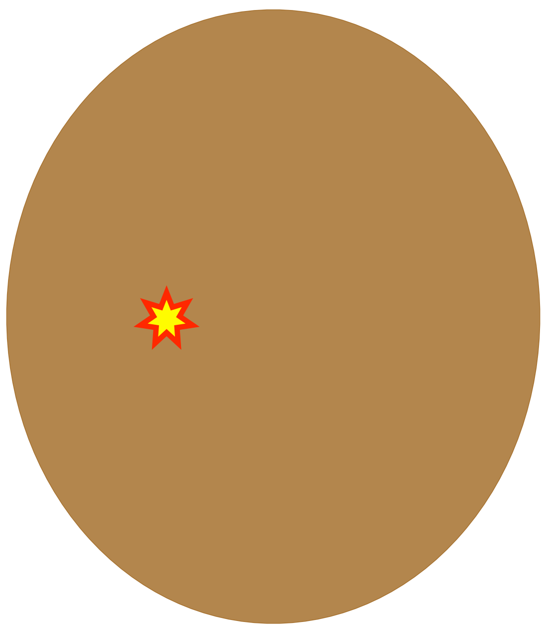

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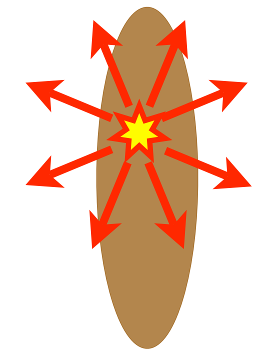

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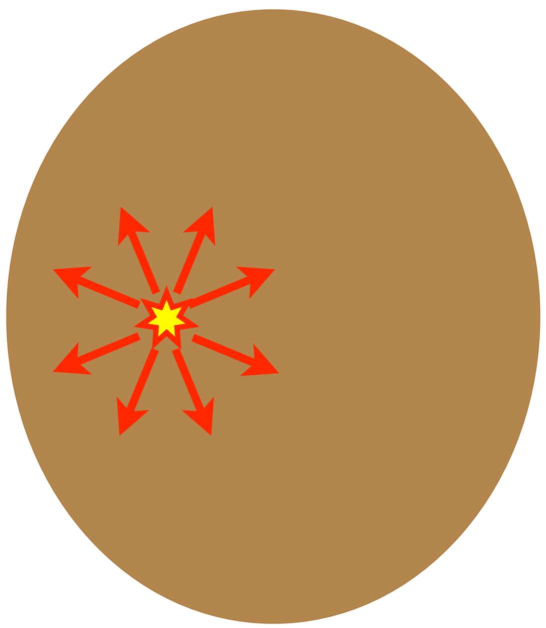
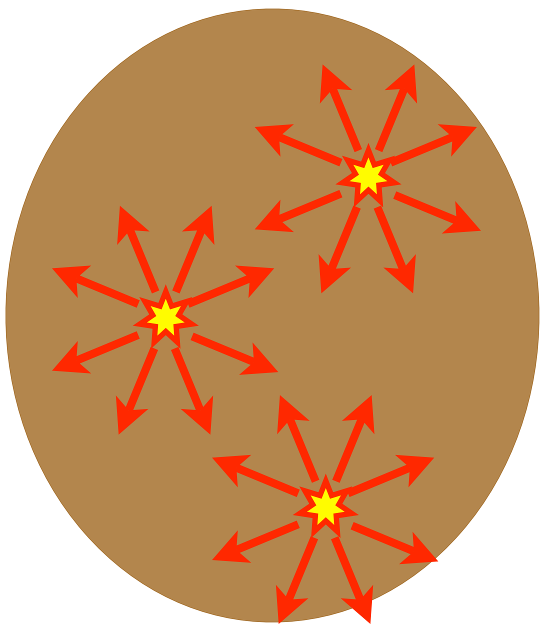


Now consider an "elementary" nucleon-nucleon collision...

A priori (no polarization!), it results in average locally in an isotropic particle emission in the transverse plane: the N-N system has no knowledge of the A-A impact parameter.

Transverse emission of particles

Each N-N collision leads locally to an emission of particles that is isotropic in the transverse plane:



If the particles thus produced do NOT subsequently interact with each other (no "final-state interaction"), the resulting emission pattern is just an incoherent sum, and it is isotropic as well:

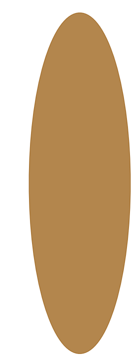
no collectivity

What if there are final state interactions?

Transverse emission of particles

Idea: the more "final-state" collisions a particle undergoes, the more its final direction (given by $\mathbf{p}_T$) deviates from its initial one.
(This should be quantified! pQCD / hadronic cross-sections...)

- overlap region in a "peripheral" collision:



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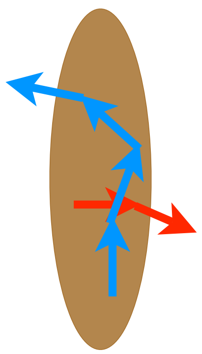

The particles emitted, in an "initial" N-N collision, along the direction of the nucleus-nucleus impact parameter (= here horizontally) undergo in average less collisions than those emitted perpendicular to it.

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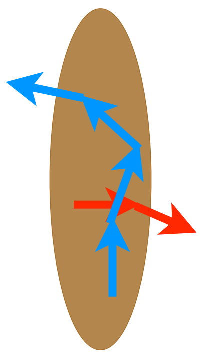

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In the end, one expects more particles with $\mathbf{p}_T$ along the A-A impact parameter ("in the reaction plane", "in-plane") than perpendicular to it ("out-of-plane"):

anisotropic emission, with a preferred axis

Highly non-trivial!!! Collective behavior at play!

Transverse emission of particles

In peripheral nucleus-nucleus collisions, the transverse-momentum distribution of outgoing particles $\frac{\mathrm{d}N}{\mathrm{d}\mathbf{p}_T}$ is anisotropic.

$$\neq$$

In a perfectly central* ($b = 0$) nucleus-nucleus collision of spherical nuclei, there is azimuthal symmetry: the $\mathbf{p}_\mathrm{T}$ distribution of outgoing particles is isotropic in the transverse plane.

⇒ The amount of anisotropy must depend on the collision centrality!

* inaccessible experimentally... yet the reasoning remains valid.

Transverse emission of particles

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How can we quantify this anisotropy?

Preferred axis: orientation of the reaction plane Φ_R.

☞ Fourier expansion (we have a 2π-periodic function!)

$$\frac{\mathrm{d}N}{\mathrm{d}\mathbf{p}_T} = \frac{\mathrm{d}N}{2\pi p_T\,\mathrm{d}p_T}\left[1 + 2v_1\cos(\varphi - \Phi_R) + 2v_2\cos 2(\varphi - \Phi_R) + \cdots\right]$$

with $v_n = \langle \cos n(\varphi - \Phi_R)\rangle$. ← average over particles

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Why no sin term in the expansion? Because there is symmetry with respect to the reaction plane: $\dfrac{\mathrm{d}N}{\mathrm{d}(\varphi - \Phi_R)}$ is even.

Anisotropic (collective) flow

$$\frac{\mathrm{d}N}{\mathrm{d}\mathbf{p}_T} = \frac{\mathrm{d}N}{2\pi p_T\,\mathrm{d}p_T}\left[1 + 2v_1\cos(\varphi - \Phi_R) + 2v_2\cos 2(\varphi - \Phi_R) + \cdots\right]$$

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- The anisotropic emission of particles in the final state is referred to as anisotropic (collective) flow,

(bad denomination: "flow" $\not\Rightarrow$ expansion of a fluid)

- and the Fourier coefficients are called "flow coefficients":
  - (v_1: "directed flow". In collisions of identical nuclei, it vanishes at mid-rapidity for symmetry reasons.)
  - v_2: "elliptic flow". The largest component at ultra-relativistic energies.

Anisotropic (collective) flow

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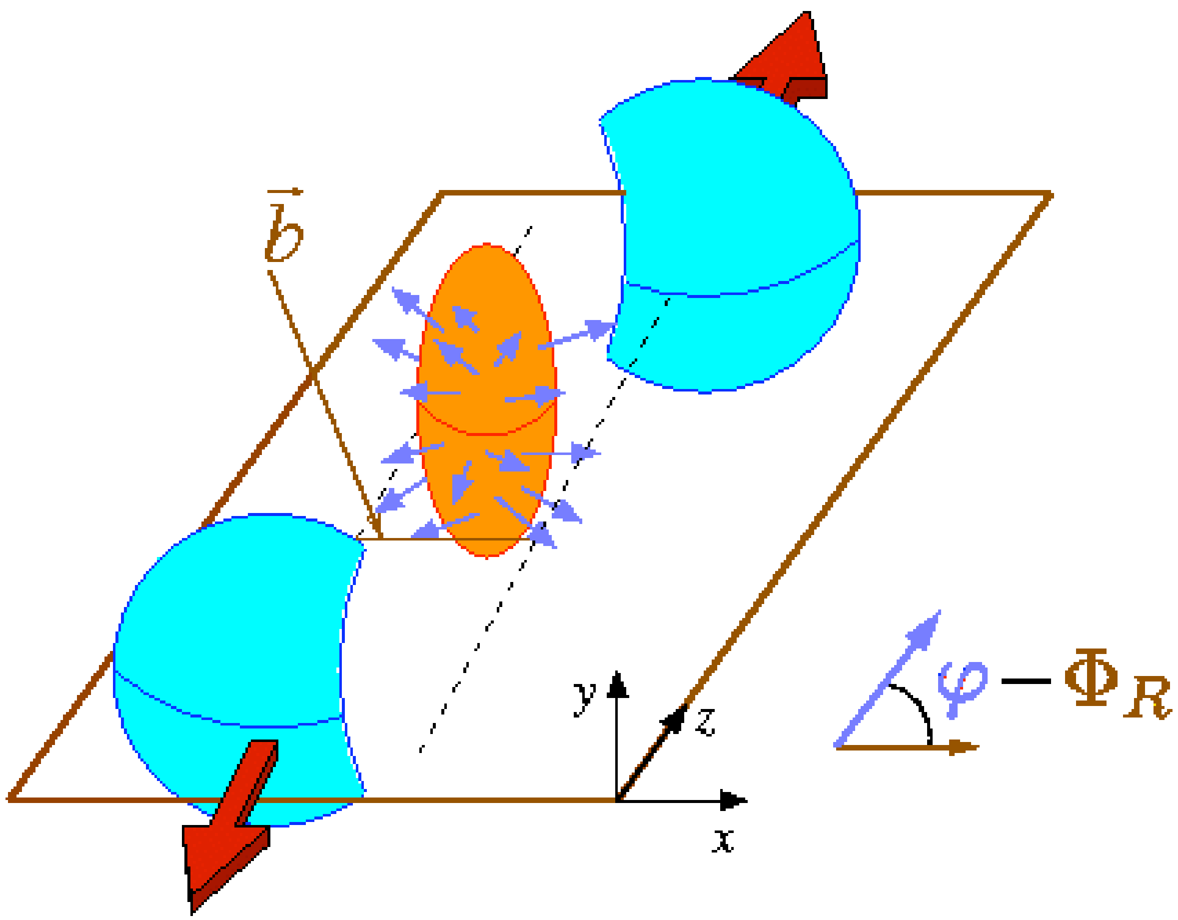
    $v_2 = \langle \cos 2(\varphi - \Phi_R)\rangle$: more particles emitted along the reaction plane ($\varphi - \Phi_R$ = 0 or 180°) than perpendicular to it ☞ $v_2 > 0$.

Anisotropic (collective) flow

Alternate, macroscopic picture

Consider a non-central collision:



anisotropy (in position space) of the source (in the plane transverse to the beam)

⇒ anisotropic pressure gradients (larger along the impact parameter)

Anisotropic (collective) flow

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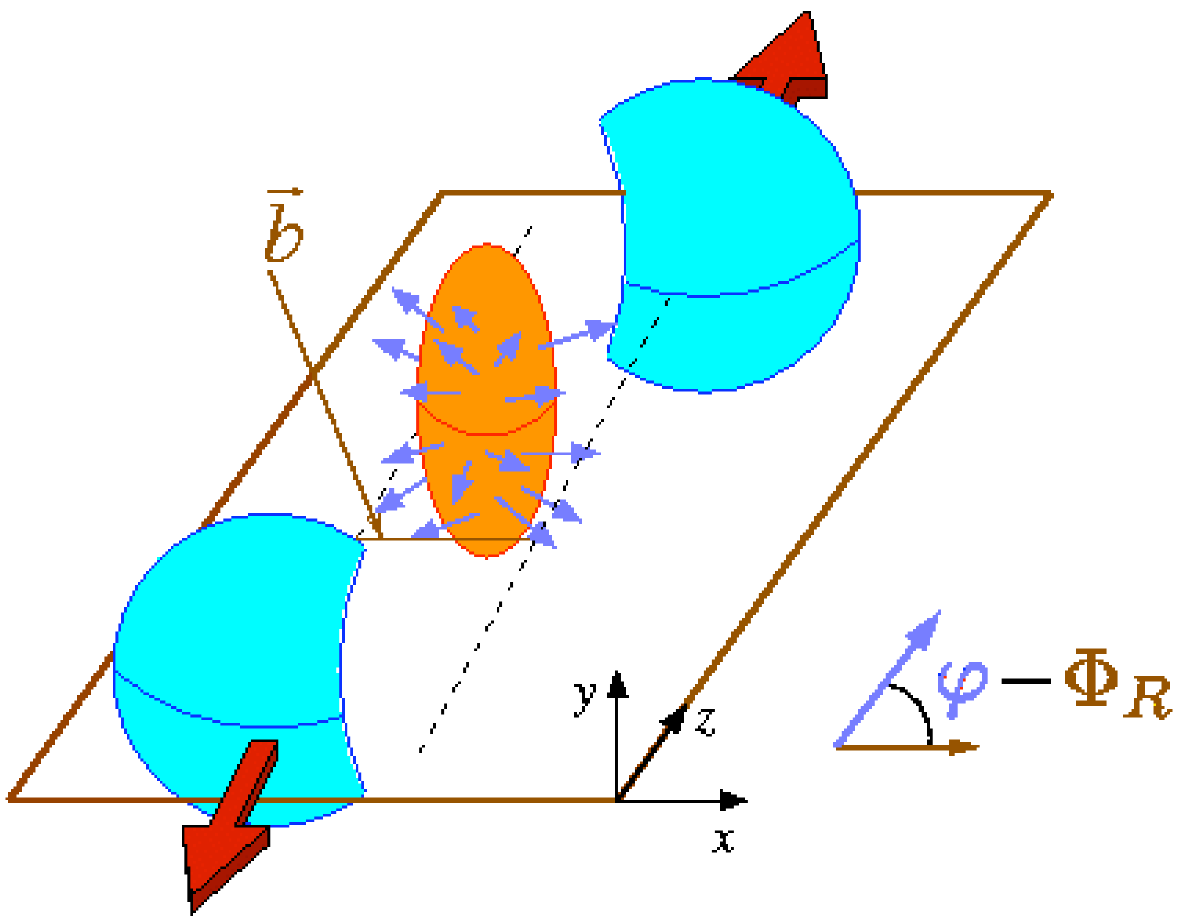

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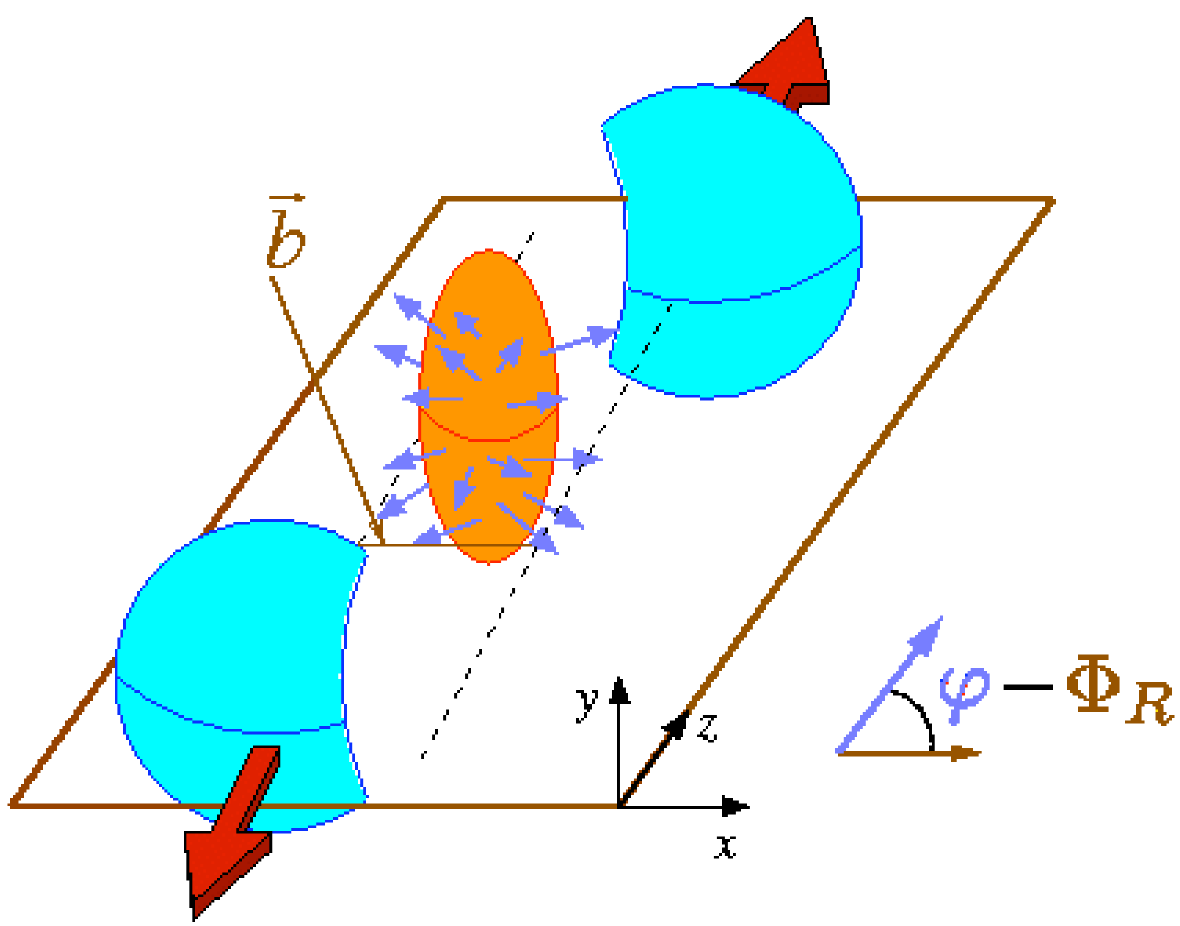

anisotropy (in position space) of the source (in the plane transverse to the beam)

⇒ anisotropic pressure gradients (larger along the impact parameter) — push

⇒ anisotropic fluid velocities

anisotropic emission of particles (in momentum space):

"anisotropic collective flow"

Anisotropic (collective) flow

Remarks

- In the above pictures, I have implicitly assumed that the remnants of the colliding nuclei have left the interaction (overlap) region, and thus do not block the emission of particles along the reaction plane.

This is true at ultra-relativistic energies, not at lower energies... where v_2 is negative because of this blocking!
(this is a rare instance where one "sees" the effect of the Lorentz contraction: the longitudinal size of the nuclei is $2R/\gamma$)

- All the arguments presented are p_T-dependent... and so are the Fourier flow coefficients: $v_2(p_T)$...

- ... not to mention the dependence on the particle type!

Anisotropic flow is a collective effect

Physical assumption ($\mathcal{A}$):

For a fixed orientation of the reaction plane Φ_R and a fixed value of the impact parameter b, each particle in the system is correlated only to a small number of particles.

Moreover, this number does not vary strongly with nuclear size and impact parameter.

Reasonable assumption (emission of "clusters" = resonances, jets...).

Except in the vicinity of a second-order phase-transition!

[I.e., keep the existence of this assumption in a corner of your mind when studying anisotropic flow at FAIR... or at "low" SPS energies(?)]

Anisotropic flow is a collective effect

Under assumption (A), define a "generating function":

$$G(x) \equiv \left\langle \prod_{j=1}^{M} (1 + x \cos\varphi_j) \right\rangle$$

where the product runs over all (detected) particles.

Angular brackets denote an average over an infinite number of events with the same centrality.

In the following, such averages will be more conveniently performed in two successive steps:

- I shall first average over events with the same impact parameter orientation; the corresponding averages will be denoted by $\langle \cdots | \Phi_R \rangle$.
- Then I average over the reaction-plane orientation, assuming that the distribution of is Φ_R isotropic.

Anisotropic flow is a collective effect

Consider first events with a fixed orientation of the impact parameter.
According to hypothesis (*A*) above, each event can be split into N independent subsystems. One may then write

$$\prod_{j=1}^{M}(1+x\,\cos\varphi_j)=\prod_{k=1}^{N}\prod_{j_k}(1+x\cos\varphi_{j_k})\,,$$

where I have factorized the product in the left-hand side into the product of the contributions of each subsystem.

The fixed-Φ_R average is then straightforward:

$$\left\langle\prod_{j=1}^{M}(1+x\,\cos\phi_j)\,|\Phi_R\right\rangle=\prod_{k=1}^{N}\left\langle\prod_{j_k}(1+x\cos\phi_{j_k})\,|\Phi_R\right\rangle$$

Anisotropic flow is a collective effect

Taking the logarithm:

$$\ln \left\langle \prod_{j=1}^{M} (1 + x \cos\phi_j) \,|\Phi_R \right\rangle = M \left(ax + bx^2 + cx^3 + \cdots\right)$$

where I have used the fact that the number N of independent subsystems scales like the total multiplicity M.

Thanks to hypothesis (*A*) the coefficients a, b, ... in this expansion are independent of the system size.

Anisotropic flow is a collective effect

cumulants

Cumulants are generally defined as the coefficients in the expansion in power series of the logarithm of the generating function.

Accordingly, let me define the cumulants corresponding to the generating function introduced above:

$$\ln G(x) \equiv \sum_{k=1}^{+\infty} \frac{x^k}{k!} c\{k\}$$

where $c\{k\}$ denotes the cumulant to order k.

Cumulants scale differently with the multiplicity according to whether (collective) anisotropic flow is present in a system or not!

Anisotropic flow is a collective effect

cumulants

Assume first that there is no anisotropic flow in the system. That is, outgoing particles are not correlated to the reaction plane: fixed-Φ_R averages are in fact independent of Φ_R, so that

$$\ln \left\langle \prod_{j=1}^{M} (1 + x \cos\phi_j) \,|\Phi_R \right\rangle = \ln G(x)$$

that is

$$\sum_{k=1}^{+\infty} \frac{x^k}{k!} c\{k\} = M \left(ax + bx^2 + cx^3 + \cdots\right)$$

so that the cumulants $c\{k\}$ scales linearly with the multiplicity M for any order k.

Anisotropic flow is a collective effect

cumulants

Consider now collisions with anisotropic flow.

Assuming $x \ll 1$,

$$\ln\left\langle \prod_{j=1}^{M} (1 + x\,\cos\phi_j)\,|\Phi_R \right\rangle \simeq \left\langle \sum_{j=1}^{M} \cos\phi_j |\Phi_R \right\rangle x = M v_1 \cos\Phi_R\, x$$

so that $\left\langle \prod_{j=1}^{M} (1 + x\,\cos\phi_j)\,|\Phi_R \right\rangle = \mathrm{e}^{M v_1 \cos\Phi_R\, x}$

which can easily be averaged over Φ_R to yield

$$G(x) = \int_0^{2\pi} \frac{\mathrm{d}\Phi_R}{2\pi} \left\langle \prod_{j=1}^{M} (1 + x\,\cos\phi_j)\,|\Phi_R \right\rangle = I_0(M v_1 x)$$

from where one can extract the successive cumulants: $c\{k\}$ scales M^k.

Anisotropic flow is a collective effect

Lee-Yang zeroes

When there is no anisotropic flow in the system, one can readily average over Φ_R : the generating function factorizes into the product of generating functions for (independent) subsystems.
This means than the positions of the zeroes of $G(x)$ do not depend on the multiplicity M.

In the presence anisotropic flow, the generating function is given by

$$G(x) = I_0(M v_1 x)$$

☞ the positions of the zeroes of $G(x)$ now obviously depend on the multiplicity M !

Analogy: Lee-Yang zeroes

Phys. Rev. 87 (1952) 404: a theory of phase transitions

- Grand partition function (T, V fixed): $\mathcal{Q}(\mu) = \sum_{N=0}^{+\infty} Z_N \mathrm{e}^{\mu N/kT}$
- Take a reference value μ_c, define $z \equiv (\mu - \mu_c)/kT$
- Let $\mathcal{G}(z) \equiv \dfrac{\mathcal{Q}(\mu)}{\mathcal{Q}(\mu_c)} = \sum_{N=0}^{+\infty} P_N \mathrm{e}^{zN}$: generating function
  - (P_N: probability to have N particles at $\mu = \mu_c$)
- Let the volume V (= the system size) increase
  - if no phase transition, the zeroes are unchanged
  - if phase transition at $\mu = \mu_c$, the zeroes of $\mathcal{G}$ come closer to 0

phase transition → long-range correlations, collective behavior

Anisotropic flow → long-range correlations, collective behavior

Anisotropic (collective) flow

Anisotropic flow requires final-state interactions to develop...

An exact computation of the dependence of v_2, v_4 on the number Kn^{-1} of collisions undergone by particles requires a microscopic transport model, yet one can guess the general tendency.

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- In the absence of rescatterings ("gas"), no flow develops.

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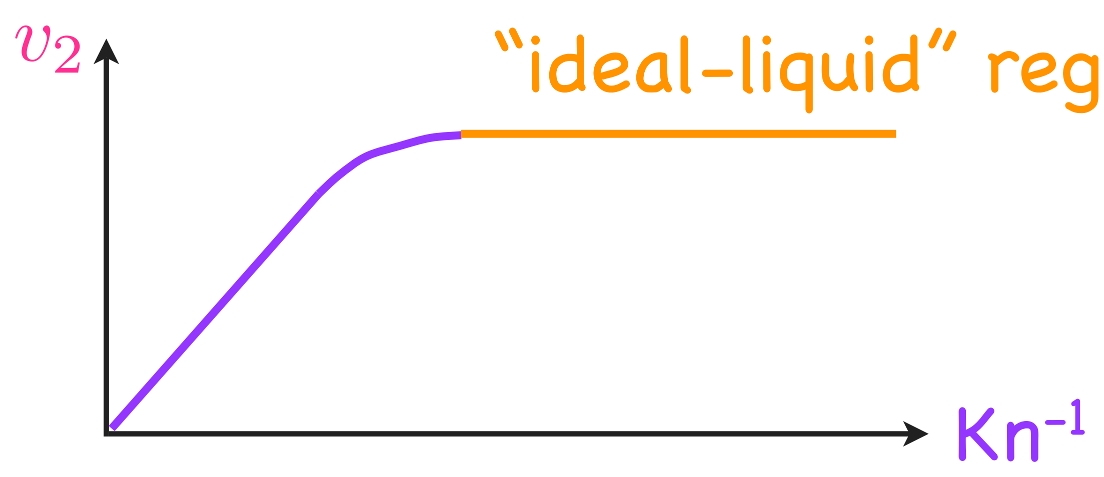
- In the absence of rescatterings ("gas"), no flow develops.
- The more collisions, the larger the flow.
- For a given number of collisions, the system thermalizes: further collisions no longer increase v_2.



Anisotropic (collective) flow

Possible experimental control knobs for the mean number of collisions per particle Kn^{-1}:

- centrality of the collisions
- size of the colliding nuclei
- center-of-mass energy of the collisions
- transverse momentum / rapidity of the emitted particles

Anisotropic (collective) flow

- The combination of the initial anisotropy in position space of the particle-emitting source and of final-state interactions between emitted particles leads to an anisotropic emission of particles: anisotropic collective flow, quantified in terms of coefficients v_1, v_2...

- It is a genuinely collective phenomenon: all particles are correlated to the reaction plane.

  ☞ reflected in the behavior of well-chosen generating functions: scaling of the cumulants / position of the zeroes with multiplicity.

- Measurements of anisotropic flow yield information on

  - at the microscopic level: in-medium cross-sections;
  - at the macroscopic level: equation of state of the created matter.

Anisotropic (collective) flow

RHIC phenomenology (abridged!)

Based on measurements of elliptic flow v_2, the creation of a "perfect liquid" was claimed.

(still a disputed issue!)

Idea: measured strength of v_2 is consistent with computations within ideal fluid dynamics (i.e., no viscosity) with an equation of state close to the expected one.