Tutorial sheet 2 (supplement)

6. Yet another example of motion of a deformable continuous medium

Consider the motion defined in a system of Cartesian coordinates with basis vectors $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ by the velocity field with components

$$\mathsf{v}^1(t, \vec{r}) = f_1(t, x^2), \quad \mathsf{v}^2(t, \vec{r}) = f_2(t, x^1), \quad \mathsf{v}^3(t, \vec{r}) = 0,$$

with f_1, f_2 two continuously differentiable functions.

Compute the strain rate tensor $\mathbf{D}(t, \vec{r})$ for this motion. What is the volume expansion rate? Give the rotation rate tensor $\mathbf{R}(t, \vec{r})$ and the vorticity vector. Under which condition(s) on the functions f_1, f_2 does the motion become irrotational?