Tutorial sheet 6

16. Two-dimensional potential flow. Teapot effect

Consider a steady two-dimensional potential flow with velocity $\vec{\mathsf{v}}(x,y)$, with (x,y) Cartesian coordinates. The associated complex velocity potential is denoted $\phi(z)$, where $z = x + \mathrm{i}y$.

i. Consider the complex potential $\phi(z) = Az^n$ with $A \in \mathbb{R}$ and $n \geq 1/2$. Show that this potential allows you to describe the flow velocity in the sector $\widehat{\mathcal{E}}$ delimited by two walls making an angle $\alpha = \pi/n$.

ii. What can you say about the flow velocity in the vicinity of the end-corner of the sector $\widehat{\mathcal{E}}$?

Hint: Distinguish the cases $\alpha < \pi$ and $\alpha > \pi$.

iii. Teapot effect

If one tries to pour tea "carefully" from a teapot, one will observe that the liquid will trickle along the lower side of the nozzle, instead of falling down into the cup waiting below. Explain this phenomenon using the flow profile introduced above (in the case $\alpha > \pi$) and the Bernoulli equation.

Literature: Jearl Walker, Scientific American, Oct. 1984 (= Spektrum der Wissenschaft, Feb. 1985).

iv. Assuming now that you are using the potential $\phi(z) = Az^n$ to model the flow of a river, which qualitative behavior can you anticipate for its bank?

17. Pointlike vortex

Consider the motion defined in a system of cylindrical coordinates by the velocity field given for $r \neq 0$ by

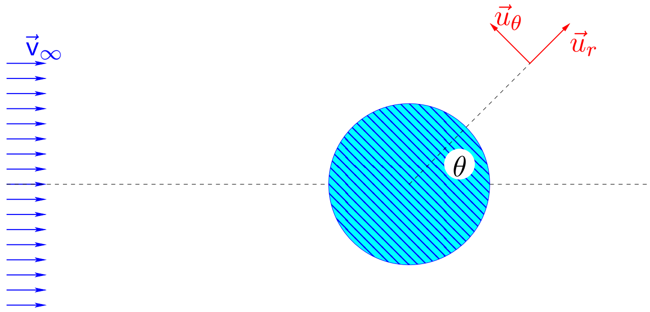
$$\vec{\mathsf{v}}(t,r,\theta,z) = \frac{\Gamma}{2\pi r^2}\vec{\mathrm{e}}_\theta, \quad \Gamma \in \mathbb{R}, \tag{1}$$

where $\vec{\mathrm{e}}_\theta$ is the natural basis vector associated with the polar angle θ.

Give the strain rate tensor, with its principal axes and eigenvalues, the volume expansion rate, the rotation rate tensor and the vorticity vector. Compute the circulation of the velocity field along a closed curve circling the z-axis.

18. Potential flow with a vortex. Magnus effect

The purpose of this exercise is to introduce a simplified model for the Magnus effect, which was discussed in the lectures.



One can show that the flow velocity of an incompressible perfect fluid around a cylinder of radius R at rest, with the uniform condition $\vec{\mathsf{v}}(\vec{r}) = \vec{\mathsf{v}}_\infty$ far from the cylinder—$\vec{\mathsf{v}}_\infty$ being perpendicular to the cylinder axis—, is given by

$$\vec{\mathsf{v}}(r,\theta) = \mathsf{v}_\infty\left[\left(1 - \frac{R^2}{r^2}\right)\cos\theta\,\vec{u}_r - \left(1 + \frac{R^2}{r^2}\right)\sin\theta\,\vec{u}_\theta\right], \tag{2}$$

where (r,θ) are polar coordinates—the third dimension (z), along the cylinder axis, plays no role—with the origin at the center of the cylinder (see Figure) and $\vec{u}_r$, $\vec{u}_\theta$ unit length vectors.

One superposes to the velocity field (2) a vortex with circulation Γ, corresponding to a flow velocity

$$\vec{\mathsf{v}}(r,\theta) = \frac{\Gamma}{2\pi r}\vec{u}_\theta. \tag{3}$$

i. Let $C \equiv \Gamma/(4\pi R \mathsf{v}_\infty)$. Determine the points with vanishing velocity for the flow resulting from superposing (2) and (3).

Hint: Distinguish the two cases $C < 1$ and $C > 1$.

ii. How do the streamlines look like in each case? Comment on the physical meaning of the result.

iii. Express the force per unit length $\mathrm{d}\vec{F}/\mathrm{d}z$ exerted on the cylinder by the flow (2)+(3) as function of Γ, v_∞ and the mass density ρ of the fluid.