Tutorial sheet 3

Discussion topics:

- What are the strain rate tensor, the rotation rate tensor, and the vorticity vector? How do they come about and what do they measure?
- What is the Reynolds transport theorem (and its utility)?
(- Give the basic equations governing the dynamics of perfect fluids.)

7. Example of a motion

Consider the motion defined in a system of Cartesian coordinates with basis vectors $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ by the velocity field with components

$$\mathsf{v}^1(t, \vec{r}) = f_1(t, x^2), \quad \mathsf{v}^2(t, \vec{r}) = f_2(t, x^1), \quad \mathsf{v}^3(t, \vec{r}) = 0,$$

with f_1, f_2 two continuously differentiable functions.

i. Compute the strain rate tensor $\mathbf{D}(t, \vec{r})$ for this motion. What are its principal axes and the corresponding eigenvalues?[1] What is the volume expansion rate?

ii. Give the rotation rate tensor $\mathbf{R}(t, \vec{r})$ and the vorticity vector. Under which condition(s) on the functions f_1, f_2 does the motion become irrotational?

8. Pointlike source

Consider the fluid motion defined in a system of cylindrical coordinates (r, θ, z) by the velocity field given for $r \neq 0$ by

$$\mathsf{v}^r(t, \vec{r}) = \frac{f(t)}{r}, \quad \mathsf{v}^\theta(t, \vec{r}) = 0, \quad \mathsf{v}^z(t, \vec{r}) = 0,$$

with f some scalar function.

i. Calculate the strain rate tensor; what are its principal axes? Give the volume expansion rate. Compute the vorticity vector.

ii. Mathematically, the velocity field is singular at $r = 0$. Thinking of the velocity profile, what do you have *physically* at that point if $f(t) > 0$? if $f(t) < 0$?

9. Pointlike vortex

Consider now the motion defined in a system of cylindrical coordinates by the velocity field given for $r \neq 0$ by

$$\vec{\mathsf{v}}(t, r, \theta, z) = \frac{\Gamma}{2\pi r}\vec{e}_\theta, \quad \Gamma \in \mathbb{R}.$$

Give the strain rate tensor, with its principal axes and eigenvalues, the volume expansion rate, the rotation rate tensor and the vorticity vector. Compute the *circulation* of the velocity field along a closed curve circling the z-axis.

[1] Need a reminder on these notions? Check your favorite lecture on the mechanics of rigid bodies, especially the chapter on the tensor of inertia: e.g. `http://www.physik.uni-bielefeld.de/~yorks/theo1/` (Nov.11 & 12 lectures) or `http://www.physik.uni-bielefeld.de/~laine/klassisch/` (Nov.12 lecture).