Simple flows of a nonrelativistic fluid

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Vortex dynamics in a perfect fluid

Circulation of the velocity field along a closed curve $\vec{\gamma}(t,\lambda)$:

$$\Gamma_{\vec{\gamma}}(t) \equiv \oint_{\vec{\gamma}} \vec{\mathsf{v}}(t,\vec{\gamma}(t,\lambda)) \cdot \mathrm{d}\vec{\ell}$$

Stokes theorem: $\Gamma_{\vec{\gamma}}(t)$ = flux of the vorticity field $\vec{\nabla}\times\vec{\mathsf{v}}(t,\vec{r})$ through the surface $\mathcal{S}_{\vec{\gamma}}(t)$ delimited by $\vec{\gamma}(t,\lambda)$.

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Kelvin's circulation theorem:

In a perfect barotropic* fluid with conservative volume forces, the circulation of the flow velocity around a closed curve (enclosing a simply connected region) comoving with the fluid is conserved.

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In a perfect barotropic* fluid with conservative volume forces

$$\frac{\mathrm{D}\Gamma_{\vec{\gamma}}(t)}{\mathrm{D}t} = 0$$

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In a perfect barotropic* fluid with conservative volume forces, the circulation of the flow velocity around a closed curve (enclosing a simply connected region) comoving with the fluid is conserved.

In a perfect barotropic fluid with conservative volume forces, the flux of the vorticity across a material surface is conserved.

Lagrange's theorem:

In a perfect barotropic fluid with conservative volume forces, if the flow is irrotational at a given instant t, it remains irrotational at later times.

* i.e. $\mathcal{P} = \mathcal{P}(\rho)$.