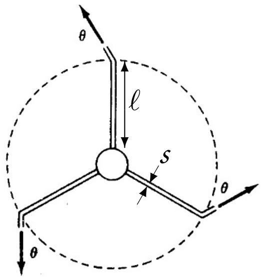
Tutorial sheet 5

Discussion topic: What is the Bernoulli equation? Give some examples of application.

8. Water sprinkler

The horizontal lawn sprinkler schematized below is fed water through its center with a mass flow rate Q. Assuming that water is a perfect incompressible fluid, determine the steady rotation rate as function of Q, the cross section area s of the pipes, their length ℓ, and the angle θ of the emerging water jets with respect to the respective pipes.



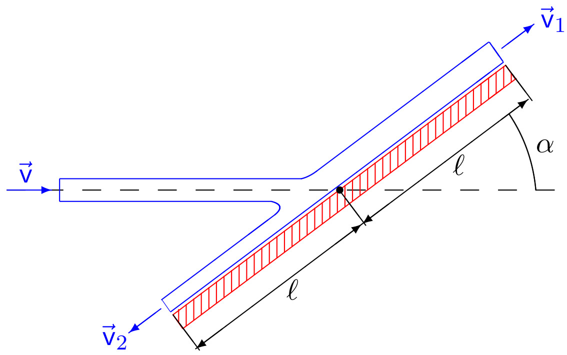
9. Rotating fluid in a uniform gravitational potential

Consider a perfect fluid contained in a straight cylindrical vessel which rotates with constant angular velocity $\vec{\Omega} = \Omega\,\vec{e}_3$ about its vertical axis, the whole system being placed in a uniform gravitational field $-g\,\vec{e}_3$. Assuming that the fluid rotates with the same angular velocity and that its motion is incompressible, determine the shape of the free surface of the fluid.

Hint: Despite the geometry, working with Cartesian coordinates is quite straightforward. At the free surface, the fluid pressure is constant (it equals the atmospheric pressure).

10. Water jet

A horizontal jet of water with cross section area $\mathcal{S} = 20$ cm^2 and velocity $\mathsf{v} = 20\,\mathrm{m\cdot s^{-1}}$ hits an inclined board of length $2\ell = 20$ cm making an angle α with the horizontal direction, and splits into two jets 1 and 2. The resulting flow is assumed to be steady and incompressible, and water is modeled as a perfect fluid.



i. Show that the influence of gravity on the velocities v_1, v_2 is negligible, so that you can forget it when applying the equation appropriate for the flow under study (which you should apply at the water/air boundary).

ii. Knowing that the force $\vec{F}$ exerted by the water on the board is normal to the latter (why?), determine the cross-section areas $\mathcal{S}_1$, $\mathcal{S}_2$ of the jets as functions of $\mathcal{S}$ and the angle α.

iii. Determine the force $\vec{F}$ and compute the numerical value of $|\vec{F}|$ for $\alpha = 30^{\mathrm{o}}$.