Simple flows of a nonrelativistic fluid

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- Hydrostatics of a perfect fluid

May 25

- Steady flows of a perfect fluid

- Vortex dynamics (June 1)

- Potential flows June 8

Hydrostatics of a perfect fluid: $\vec{v} = \vec{0}$

A single, simple equation: $\vec{\nabla}\mathcal{P}(\vec{r}) = \vec{f}_V(\vec{r})$

If the external volume forces are conservative* $\vec{f}_V(\vec{r}) = -\rho(\vec{r})\vec{\nabla}\Phi(\vec{r})$

$$\frac{\vec{\nabla}\mathcal{P}(\vec{r})}{\rho(\vec{r})} = -\vec{\nabla}\Phi(\vec{r})$$

☞ Leads to:

- Archimedes principle
- In a constant gravitational field $\Phi(\vec{r}) = gz$, simple solutions for an "incompressible fluid", an isothermal fluid, an "isentropic" fluid...

* the relation also holds for the centrifugal inertial force

Steady inviscid flows: $\partial_t = 0$

… still for a fluid with conservative volume forces $\vec{f}_V(\vec{r}) = -\rho(\vec{r})\vec{\nabla}\Phi(\vec{r})$

From the Euler equation, one derives the Bernoulli* equation

$$\frac{\vec{\mathrm{v}}(\vec{r})^2}{2} + \frac{w(\vec{r})}{\rho(\vec{r})} + \Phi(\vec{r}) = \text{constant along a streamline}$$

where w denotes the enthalpy density.

- In irrotational flows $\displaystyle\frac{\vec{\mathrm{v}}(\vec{r})^2}{2} + \frac{w(\vec{r})}{\rho(\vec{r})} + \Phi(\vec{r}) =$ constant everywhere.
- In incompressible flows, w can (should?) be replaced by the pressure:

$$\frac{\vec{\mathrm{v}}(\vec{r})^2}{2} + \frac{\mathcal{P}(\vec{r})}{\rho(\vec{r})} + \Phi(\vec{r}) = \text{constant along a streamline}$$

* pronounced "Bärnuji"

Applications of the Bernoulli equation

- Torricelli's law for the drainage of a vessel: like free fall!

  ☞ exercise with a clepsydra (Wasseruhr)

- Venturi effect: for an incompressible flow through a tube with varying cross section

- Pitot tube: to measure flow velocities using a manometer

- Magnus effect

  ☞ responsible for topspin, backspin or lateral slice for football / (table) tennis / golf players (or fans)

  ☞ and rotor ships