Kinematics of a continuous medium

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Kinematics of a continuous medium

- Generic motion of a fluid

- Classification of fluid flows

May 4

Throughout these slides, Cartesian coordinates ($\{x_j\}$, $\{\mathrm{v}_j\}$, ...) are used:

$$x_j = x^j \ , \ \mathrm{v}_j = \mathrm{v}^j \ldots \text{ for } j = 1,2,3$$

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Generic motion of a fluid

A small fluid element inside a larger moving fluid may undergo three types of transformations:

- **translation**: displacement of the center of mass of the fluid element

  ☞ like a point particle

- (local) **rotation**: around an axis going through the center of mass

  ☞ like a rigid body (non-deformable solid)

- **deformations**:
  - change of the shape at constant volume: **shear deformation**
  - change of the volume (**dilatation** ≠ **contraction**) while the shape remains constant

  ☞ only for deformable continuous media!

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Generic motion of a fluid

The three types of transformations of the fluid element at $\vec{r}$ at time t are encoded in the fluid velocity $\vec{\mathrm{v}}(t,\vec{r})$:

- translation: position shift $\delta\vec{r} = \vec{\mathrm{v}}(t,\vec{r})\,\delta t$

  ☞ like a point particle

- (local) rotation: a fluid line element $\delta\vec{\ell}$ spins with the rotation rate

$$\frac{1}{2}\vec{\omega}(t,\vec{r}) \times \delta\vec{\ell}$$

☞ vorticity field $\vec{\omega}(t,\vec{r}) \equiv \vec{\nabla} \times \vec{\mathrm{v}}(t,\vec{r})$

☞ antisymmetric components ($j \neq k$): $\dfrac{\partial \mathrm{v}^j(t,\vec{r})}{\partial x^k} - \dfrac{\partial \mathrm{v}^k(t,\vec{r})}{\partial x^j}$

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Generic motion of a fluid

The three types of transformations of the fluid element at $\vec{r}$ at time t are encoded in the fluid velocity $\vec{\mathrm{v}}(t,\vec{r})$:

- deformations: described by the symmetric elements ($j,k=1,2,3$):

$$\frac{\partial \mathrm{v}^j(t,\vec{r})}{\partial x^k} + \frac{\partial \mathrm{v}^k(t,\vec{r})}{\partial x^j} \equiv 2\mathbf{D}_{jk}(t,\vec{r})$$

☞ "deformation rate tensor"

  - change of shape at constant volume: $\mathbf{D}_{jk}(t,\vec{r})$ with $j \neq k$
  - change of volume at constant shape: $\underbrace{\mathbf{D}_{jj}(t,\vec{r})}$ with sum over j.

$$= \vec{\nabla}\cdot\vec{\mathrm{v}}(t,\vec{r})$$

(volume) expansion rate

Incompressible flow $\Leftrightarrow$ $\vec{\nabla}\cdot\vec{\mathrm{v}}(t,\vec{r}) = 0$ everywhere

Classification of fluid flows

- Geometrical criteria
  - one-dimensional, two-dimensional…
- Kinematic criteria
  - uniform ≠ non-uniform, steady ≠ unsteady
  - rotational / vortical ≠ irrotational (≡ potential!): $\vec{\omega}(t,\vec{r}) \equiv \vec{\nabla} \times \vec{v}(t,\vec{r})$
  - incompressible ≠ compressible: $\vec{\nabla} \cdot \vec{v}(t,\vec{r})$
  - subsonic ≠ supersonic: Mach number
  - laminar ≠ turbulent
- Physical criteria
  - viscous ≠ non-viscous, isothermal, isentropic…