Basic notions on continuous media

UNIVERSITÄT BIELEFELD
Fakultät für Physik

Basic notions on continuous media

- The model of a continuous medium

April 20

- Lagrangian and Eulerian descriptions
- Mechanical stress

April 27

Basic notions on continuous media

- The model of a continuous medium

April 20

- Lagrangian and Eulerian descriptions
- Mechanical stress

April 27

The model of a continuous medium

☞ A statistical model for the "large-scale" behavior of a many-body system (with $N = 10^3 - 10^{30}$ "elementary" constituents):

- Forget the discrete constituents (atoms…) and the 6N corresponding kinematic variables (positions & momenta: trajectories)

- … and describe the system as a continuum, characterized by a few fields that depend on time t and position $\vec{r}$:

  - Coarse graining: the "material point" / "fluid particle" at a given $\vec{r}$ corresponds actually to a finite but very small volume containing many (yet $\ll N$) atoms.

  - The medium properties (= densities, local intensive thermodynamic variables) should vary smoothly with $\vec{r}$.

… in general: there will be exceptions!

The model of a continuous medium

☞ A statistical model for the "large-scale" behavior of a many-body system (with N = 10³ – 10³⁰ "elementary" constituents):

- Forget the discrete constituents (atoms…) and the 6N corresponding kinematic variables (positions & momenta: trajectories)
- … and describe the system as a continuum, characterized by a few fields that depend on time t and position $\vec{r}$:
  - Coarse … a given $\vec{r}$ correspon… containing many (yet…
  - The medium properties (= densities, local intensive thermodynamic variables) should vary smoothly with $\vec{r}$.

… in general: there will be exceptions!

Does it make sense to describe 1000 particles as a continuous medium?? Why not 100? Or 10?

A timely question!

International Workshop on Discovery Physics at the LHC (Kruger2018) IOP Publishing
IOP Conf. Series: Journal of Physics: Conf. Series **1271** (2019) 012018 doi:10.1088/1742-6596/1271/1/012018

Hydrodynamic flow in small systems

or: "How the heck is it possible that a system emitting only a dozen particles can be described by fluid dynamics?"

Ulrich Heinz[1a], in collaboration with J. Scott Moreland[b]

[a]Department of Physics, The Ohio State University, Columbus, OH 43210-1117, USA
[b]Department of Physics, Duke University, Durham, NC 27708-0305, USA

E-mail: `heinz.9@osu.edu`

Abstract. The "unreasonable effectiveness" of relativistic fluid dynamics in describing high energy heavy-ion and even proton-proton collisions will be demonstrated and discussed. Several recent ideas of optimizing relativistic fluid dynamics for the specific challenges posed by such collisions will be presented, and some thoughts will be offered why the framework works better than originally expected. I will also address the unresolved question where exactly hydrodynamics breaks down, and why.

Local thermodynamic equilibrium

At each point $\vec{r}$ of the continuous medium under study, the equation of state — e.g.: relation between pressure, temperature, and number density — is the same as in a system with the same microscopic constituents in the thermodynamic limit.

☞ should generally be ensured in the limit of very small Knudsen numbers

$$\mathrm{Kn} \equiv \frac{\ell_{\rm mfp}}{L}$$

$\ell_{\rm mfp}$: mean free path

L: macroscopic length scale

Basic notions on continuous media

- The model of a continuous medium

April 20

- Lagrangian and Eulerian descriptions
- Mechanical stress

April 27

Describing the motion of a continuous medium

- Lagrangian description
  - Variables $t, \vec{R}$ (in the initial / reference configuration)
  - Follow the trajectories $\vec{r} = \vec{r}(t, \vec{R})$ of the material points
    - ☞ material point velocity $\vec{v}(t) = \dfrac{\partial \vec{r}(t, \vec{R})}{\partial t}$

Equivalent descriptions!

- Eulerian description
  - Variables $t, \vec{r}$ (time-independent); fields $\mathcal{G} = \mathcal{G}(t, \vec{r})$
  - Velocity field: $\vec{\mathrm{v}}(t, \vec{r})$ = velocity $\vec{v}(t)$ of the material point that goes through $\vec{r}$ at time t

Describing the motion of a continuous medium

- Lagrangian description
  - Pathlines (= trajectories), streaklines
    - ☞ can easily be imaged

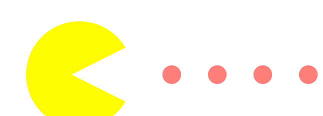
- Eulerian description
  - Streamlines (= field lines of the velocity field $\vec{\mathrm{v}}(t,\vec{r})$ at a given t)
    - ☞ mathematical construction!

Material derivative (of a field)

$$\frac{\mathrm{D}}{\mathrm{D}t} \equiv \frac{\partial}{\partial t} + \vec{\mathrm{v}}(t,\vec{r}) \cdot \vec{\nabla}$$

⚠️ shorthand notation: $\equiv \dfrac{\partial}{\partial t} + \mathrm{v}_x(t,\vec{r})\dfrac{\partial}{\partial x} + \mathrm{v}_y(t,\vec{r})\dfrac{\partial}{\partial y} + \mathrm{v}_z(t,\vec{r})\dfrac{\partial}{\partial z}$

⚠️ do not confuse $\vec{\mathrm{v}}(t,\vec{r}) \cdot \vec{\nabla}$ with the divergence $\vec{\nabla} \cdot \vec{\mathrm{v}}(t,\vec{r})$!

⚠️ differential operator: needs to act on a field... 

Material derivative (of a field)

$$\frac{\mathrm{D}}{\mathrm{D}t} \equiv \frac{\partial}{\partial t} + \vec{\mathrm{v}}(t,\vec{r}) \cdot \vec{\nabla}$$

☞ Measures the rate of change of a (Eulerian) quantity $\mathcal{G}(t,\vec{r})$...

- due to the non-steadiness ($\partial_t \neq 0$) of the motion at point $\vec{r}$

  "local derivative"

- due to the transport of matter encoded in $\vec{\mathrm{v}}(t,\vec{r})$

  "convective derivative"

Forces in a continuous medium

Two types of forces on material element at $\vec{r}$ in a continuous medium:

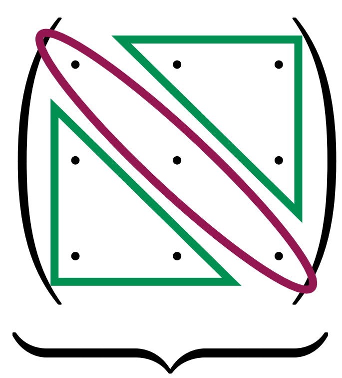
- **Volume forces**:
  - long-range (e.g.: gravity, inertial forces)
  - proportional to the volume of the material element
  - ☞ characterized by a volume density $\vec{f}_V(\vec{r})$.
- **Surface forces**:
  - due to the neighboring material elements: contact forces
  - acting at each point of the material-element surface
  - ☞ characterized by a stress vector $\vec{T}_s(\vec{r})$ ≡ force per unit area
  - normal stress resp. shear stress: orthogonal resp. tangential to the surface element.

Forces in a continuous medium

stress vector $\vec{T}_s(\vec{r})$ ≡ force per unit area

☞ depends on the orientation of the surface element acted upon

unit normal vector

$\vec{T}_s(\vec{r}) =$  $\vec{\mathrm{e}}_n$

Cauchy stress tensor $\boldsymbol{\sigma}(\vec{r})$

- diagonal elements: yield the normal stress
- off-diagonal elements (rem.: symmetric): yield the shear stress

Fluids

... are continuous media that keep deforming as long as they undergo shear stresses.

☞ gases, liquids, plasmas (+ human or animal crowds...)

In a fluid at rest, there can be no shear stress, only normal stresses (a.k.a. pressure)

⇒ diagonal stress tensor $\sigma^i{}_j(\vec{r}) = -\mathcal{P}(\vec{r})\,\delta^i{}_j$