Tutorial sheet 9

Discussion topic: Kinetic theory: which object does it consider? How does the fundamental equation look like? Which assumptions on the system underlie the description?

Throughout the exercise sheet, a system of units such that the constants c, $\hbar$ and k_B all equal 1 is used.

18. A possible scaling of the anisotropic flow coefficients of composite particles

Consider heavy-ion collisions resulting in the emission of protons and neutrons with the same transverse-momentum dependent anisotropic flow coefficients $v_{2,N}(p_T)$, ..., where the subscript N stands for nucleons. If a proton and neutron fly together — that is, with same transverse velocity or equivalently momentum — along the same direction for long enough, they may bind together into a deuteron (^{2}H). If three nucleons fly together, they may form a ^{3}H or ^{3}He nucleus.

Assuming that these light nuclei really result from the "coalescence" or 2 resp. 3 nucleons with the same transverse momentum, can you express their respective elliptic flow coefficients $v_{2,{}^2\mathrm{H}}(p_T)$, $v_{2,{}^3\mathrm{H}}(p_T)$, $v_{2,{}^3\mathrm{He}}(p_T)$ as a function of the elliptic flow $v_{2,N}$ of nucleons at an appropriate transverse momentum? Give $v_{2,{}^2\mathrm{H}}(p_T)$, resp. $v_{2,{}^3\mathrm{H}}(p_T)$ and $v_{2,{}^3\mathrm{He}}(p_T)$ up to second resp. third order in $v_{2,N}$.

Hint: You may want to go back to the level of the transverse momentum ($\mathbf{p}_T$) distributions.

19. Thermodynamics of a relativistic gas of massive particles

Consider a gas of particles of mass m with a phase-space distribution given by the Maxwell–Jüttner distribution[1]

$$f(\mathsf{x},\mathsf{p}) = C\,\mathrm{e}^{p^\mu u_\mu/T}, \tag{1}$$

where C is a normalization constant while both u_μ and T are independent of x.

i. Compute the corresponding energy-momentum tensor, i.e. (why?) the energy density ε and pressure $\mathcal{P}$ of the gas, as a function of T.

You may find that your results look nicer when expressed in terms of modified Bessel functions of the second kind (of integer order n)

$$K_n(z) = \int_0^\infty \mathrm{e}^{-z\cosh\lambda}\cosh(n\lambda)\,\mathrm{d}\lambda,$$

where $\mathrm{Re}(z) > 0$.

Hint: Work in the appropriate reference frame!

ii. Compute the particle number density $n(T)$. How is it related to the pressure? What do you recognize?

Remark: this calculation of the particle number density actually *determines* the value of C.

iii. Using your results for $\varepsilon(T)$ and $n(T)$, compute the energy per particle $\varepsilon(T)/n(T)$ in the non-relativistic limit $T \ll m$ up to order T^2. Interpret the first two terms of your result.

Hint: You may use the asymptotic expansion (valid for $|\arg z| < \frac{3\pi}{2}$)

$$K_n(z) \underset{z\to\infty}{\sim} \sqrt{\frac{\pi}{2z}}\,\mathrm{e}^{-z}\left[1 + \frac{4n^2-1}{8z} + \frac{(4n^2-1)(4n^2-9)}{2!(8z)^2} + \mathcal{O}(z^{-3})\right].$$

Remark: The Maxwell–Jüttner distribution is for "classical" particles and ignores both Bose–Einstein enhancement and Pauli's exclusion principle.

[1] Beware! in the exponent the mostly-plus metric is used.