Tutorial sheet 6

12. A toy model of multiparticle correlations and cumulants

The purpose of this exercise is to illustrate the meaning of the cumulants of multiparticle azimuthal averages on a simple example of particle emission with two- or three-particle correlations. Throughout the exercise, $\langle\cdots\rangle$ denotes an average over particles and events (in two successive steps).

i. Two-particle correlations

Consider first a set of events, such that in each event exactly M particles are emitted as follows: an angle Ψ_R and $M/2$ other angles ϕ_j are chosen randomly with an isotropic distribution. Then for each $j = 1, 2, \ldots, M/2$, two particles (labeled j and $M/2 + j$) are emitted with the same azimuth $\varphi_j = \varphi_{M/2+j} = \phi_j$.

a) What is the value of the anisotropic flow coefficients $v_n = \left\langle \mathrm{e}^{\mathrm{i}n(\varphi_j - \Psi_R)} \right\rangle$ (for any $n \in \mathbb{N}^*$)? Same question for the single- and two-particle averages $\left\langle \mathrm{e}^{\mathrm{i}n\varphi_j} \right\rangle$ and $\left\langle \mathrm{e}^{\mathrm{i}n(\varphi_j+\varphi_k)} \right\rangle$ with $j \neq k$.

Hint: No explicit calculation is needed!

b) Compute for any $n \in \mathbb{N}^*$ the two-particle average $\left\langle \mathrm{e}^{\mathrm{i}n(\varphi_j-\varphi_k)} \right\rangle$ with $j \neq k$. (*Hint*: You may want to distinguish the two cases $|j-k| \neq M/2$ and $|j-k| = M/2$ and count how often each of these two possibilities occur in a given event).

Define the two-particle azimuthal cumulant as $c_n\{2\} \equiv \left\langle \mathrm{e}^{\mathrm{i}n(\varphi_j-\varphi_k)} \right\rangle - \left\langle \mathrm{e}^{\mathrm{i}n\varphi_j} \right\rangle \left\langle \mathrm{e}^{-\mathrm{i}n\varphi_k} \right\rangle$ and a flow estimate $v_n\{2\}$ by $v_n\{2\}^2 \equiv c_n\{2\}$. What do you find for $v_n\{2\}$?

c) Compute for any $n \in \mathbb{N}^*$ the four-particle average $\left\langle \mathrm{e}^{\mathrm{i}n(\varphi_j+\varphi_k-\varphi_l+\varphi_p)} \right\rangle$ where the four particles are all different. Define the four-particle azimuthal cumulant as

$$c_n\{4\} \equiv \left\langle \mathrm{e}^{\mathrm{i}n(\varphi_j+\varphi_k-\varphi_l-\varphi_p)} \right\rangle - \left\langle \mathrm{e}^{\mathrm{i}n(\varphi_j-\varphi_l)} \right\rangle \left\langle \mathrm{e}^{-\mathrm{i}n(\varphi_k-\varphi_p)} \right\rangle - \left\langle \mathrm{e}^{\mathrm{i}n(\varphi_j-\varphi_p)} \right\rangle \left\langle \mathrm{e}^{-\mathrm{i}n(\varphi_k-\varphi_l)} \right\rangle$$

and a flow estimate $v_n\{4\}$ by $v_n\{4\}^4 \equiv -c_n\{4\}$. What do you find for $v_n\{4\}$? Can you justify the definition of $c_n\{4\}$? (why are there no extra terms?)

ii. Three-particle correlations

(optional: estimate first if you will be able to solve the following in a quarter of hour or less)

Assume now that each event consists of $M/3$ isotropically distributed triplets of particles with the same azimuth $\varphi_j = \varphi_{M/3+j} = \varphi_{2M/3+j} = \phi_j$ — generalizing the setup of question **i.**.

Repeat the calculations of questions **i.b)** and **i.c)** for the new setup.

13. Momentum conservation

Consider M particles such that the sum of their (transverse) momenta vanishes:

$$\mathbf{p}_1 + \mathbf{p}_2 + \cdots \mathbf{p}_M = \mathbf{0}. \tag{1}$$

For simplicity, we assume that all momenta have the same modulus: $|\mathbf{p}_1| = \cdots = |\mathbf{p}_M|$.

Let φ_j denote the azimuthal angle of $\mathbf{p}_j$ with respect to some fixed direction. Assuming that the azimuths are isotropically distributed — which somehow implies $M \gg 1$ to make sense — leads automatically to $\langle \cos\varphi_j \rangle = 0$. Nevertheless, and possibly contrary to your intuition, $\langle \cos(\varphi_j - \varphi_k) \rangle$ with $j \neq k$ is non-zero. More precisely, show that Eq. (1) implies

$$\langle \cos(\varphi_j - \varphi_k) \rangle \underset{M \gg 1}{\simeq} -\frac{1}{M} \tag{2}$$

and give a one-sentence physical interpretation of that result.

Hint: You may want to square Eq. (1).