Tutorial sheet 5

10. Ellipticity and quadrangularity

In exercise **7.i.** you have computed the function $\mathrm{d}^2T_{AA}(\mathbf{b})/\mathrm{d}^2\mathbf{s} \equiv T_A(\mathbf{s}-\mathbf{b}/2)T_A(\mathbf{s}+\mathbf{b}/2)$ as a function of the impact parameter (modulus) b for the collision between two lead nuclei, with $T_A(\mathbf{s})$ the nuclear thickness function. Using $\mathrm{d}^2T_{AA}(\mathbf{b})/\mathrm{d}^2\mathbf{s}$ as a weight, compute and plot as a function of b the "ellipticity" and "quadrangularity"

$$\epsilon_2^{(\mathbf{r})} \equiv -\frac{\langle r^2\cos(2\theta)\rangle}{\langle r^2\rangle} = \frac{\langle y^2-x^2\rangle}{\langle x^2+y^2\rangle} \quad , \quad \epsilon_4^{(\mathbf{r})} \equiv -\frac{\langle r^4\cos(4\theta)\rangle}{\langle r^4\rangle}, \tag{1}$$

where (x,y) resp. (r,θ) are Cartesian resp. polar coordinates with the origin at the center of the overlap region, while the impact parameter between the two Pb nuclei is assumed to be along the x-direction.

Hint: For $\epsilon_4^{(\mathbf{r})}$, it might be convenient to find the expression in Cartesian coordinates.

11. Two-particle probability distributions

In this exercise, we assume that it is possible to find a class of events in which the impact-parameter direction Ψ_R is isotropically distributed, and such that at a fixed Ψ_R the single-particle azimuthal probability distribution is given by

$$p_1(\varphi\,|\,\Psi_R) = \frac{1}{2\pi}\big(1+2v_2\cos[2(\varphi-\Psi_R)]\big), \tag{2}$$

where v_2 is independent of Ψ_R. We want to investigate two-particle probability distributions, that describe the probability to have a first particle with azimuth φ_a and a second one with azimuth φ_b.

i. Azimuthally sensitive two-particle probability distribution

In the absence of (two-body) correlation, the two-particle probability distribution is simply the product of single-particle distributions: $p_2(\varphi_a,\varphi_b\,|\,\Psi_R) = p_1(\varphi_a\,|\,\Psi_R)\,p_1(\varphi_b\,|\,\Psi_R)$. Express this probability distribution in terms of the "pair angle" $\varphi^{\rm pair} \equiv \frac{1}{2}(\varphi_a+\varphi_b)$ and the angle difference $\Delta\varphi \equiv \varphi_a-\varphi_b$.

ii. Azimuthally insensitive two-particle probability distribution

a) In most experimental analyses, the two-particle distribution $p_2(\varphi^{\rm pair},\Delta\varphi|\Psi_R)$ is integrated over $\varphi^{\rm pair}$ and averaged over Ψ_R. What do you obtain in that case? Let us denote $\mathfrak{p}(\Delta\varphi)$ the resulting distribution.

b) Can you *without calculation* tell how $\mathfrak{p}(\Delta\varphi)$ looks like if besides v_2 the single-particle probability distribution $p_1(\varphi|\Psi_R)$ also contains higher harmonics v_3, v_4, and v_5?

Plot your educated guess for $\mathfrak{p}(\Delta\varphi)$, multiplied by a factor 2π, for $-\frac{\pi}{2} \le \Delta\varphi \le \frac{3\pi}{2}$ with the values[1] $v_2 = 0.041$, $v_3 = 0.053$, $v_4 = 0.034$, and $v_5 = 0.012$ — and compare your results with figure 4 of the ALICE Collaboration article Phys. Rev. Lett. **107** (2011) 032302.[2]

iii. Azimuthally sensitive two-particle probability distribution revisited

Let us go back to question **i.**, again looking at $p_2(\varphi^{\rm pair},\Delta\varphi|\Psi_R)$ resulting from the single-particle distribution (2). Focusing on the dependence on $\varphi^{\rm pair}-\Psi_R$ at fixed $\Delta\varphi$, how could you define "pair flow coefficients" to characterize it? Which of these harmonic coefficients are non-zero in the present case? Can you give their expressions? (Beware: a mistake is quickly made!)

[1]... taken without any guarantee!

[2]You can also find the plot at `http://alice-publications.web.cern.ch/node/3879`.