Tutorial sheet 3

Discussion topic: What is the scope of the "Glauber model" introduced in the lecture?

7. Nuclear thickness and nuclear overlap functions

The nuclear density of large spherical nuclei is often approximated by the Woods–Saxon distribution

$$\rho_A(\vec{r}) = \frac{\rho_0}{1 + \mathrm{e}^{\frac{|\vec{r}|-R}{a}}} \tag{1}$$

where the values of ρ_0, R and a depend on the nucleus under consideration. For instance, $R_{\mathrm{Pb}} = 6.68$ fm and $a_{\mathrm{Pb}} = 0.546$ fm for lead or $R_{\mathrm{Au}} = 6.38$ fm and $a_{\mathrm{Au}} = 0.535$ fm for gold — the value of ρ_0, which ensures that ρ_A is normalized to A, is irrelevant in the following.

In the lecture, the nuclear thickness function was defined as ($\mathbf{s}$ denotes a transverse vector)

$$T_A(\mathbf{s}) \equiv \int \frac{\rho_A(\mathbf{s}, z)}{A}\, \mathrm{d}z \tag{2}$$

and the nuclear overlap function (for identical nuclei) at impact parameter $\mathbf{b}$ as

$$T_{AA}(\mathbf{b}) \equiv \int T_A\!\left(\mathbf{s} - \frac{\mathbf{b}}{2}\right) T_A\!\left(\mathbf{s} + \frac{\mathbf{b}}{2}\right) \mathrm{d}^2\mathbf{s} \tag{3}$$

i. Using the approach of your choice,[1] compute and plot (contour/density/3D plot? your choice!) the thickness function $T_A(\mathbf{s} = x, y)$ of Pb for $|x|, |y| \leq 10$ fm. Same question for the (nameless) function[2] $T_A(\mathbf{s} - \mathbf{b}/2)T_A(\mathbf{s} + \mathbf{b}/2)$ for the four values $|\mathbf{b}| = 0, 4, 8$ and 12 fm and in the same (x, y)-range.

ii. In the "Glauber model", the average number of binary nucleon–nucleon collisions is related to the overlap function by $\langle N_{\mathrm{coll}}^{AA}(\mathbf{b})\rangle = A^2 T_{AA}(\mathbf{b})\sigma_{\mathrm{NN}}^{\mathrm{inel}}$. Compute and plot $\langle N_{\mathrm{coll}}^{AA}\rangle$ as a function of $b = |\mathbf{b}|$ in Pb–Pb collisions for impact parameters $0 \leq b \leq 20$ fm, using $\sigma_{\mathrm{NN}}^{\mathrm{inel}} = 64$ mb.

iii. Let $\mathfrak{S}_{AA}(\mathbf{b}) \equiv 1 - \mathrm{e}^{-A^2 T_{AA}(\mathbf{b})\sigma_{\mathrm{NN}}^{\mathrm{inel}}}$. Compute and plot for $0 \leq b \leq 20$ fm the function

$$\frac{\mathrm{d}\sigma_{AA}^{\mathrm{inel}}(b)}{\mathrm{d}b} \equiv \int \mathfrak{S}_{AA}(\mathbf{b})\, b\, \mathrm{d}\varphi_{\mathbf{b}}, \tag{4}$$

where $\varphi_{\mathbf{b}}$ denotes the azimuthal angle of $\mathbf{b}$, for Pb–Pb collisions, using the same value of $\sigma_{\mathrm{NN}}^{\mathrm{inel}}$ as in **ii.** Eventually, give the value of the integral

$$\int_0^\infty \frac{\mathrm{d}\sigma_{AA}^{\mathrm{inel}}(b)}{\mathrm{d}b}\, \mathrm{d}b, \tag{5}$$

where in practice you may take 20 fm as upper bound.

If you have automatized the calculations of this exercise, you may repeat everything for other sets of parameters — and possibly another form for ρ_A — if some day you want/need to.

[1] Mathematica, C/C++, Python...

[2] A "natural" notation for this function would be $\mathrm{d}^2T_{AA}(\mathbf{b})/\mathrm{d}^2\mathbf{s}$, cf. Eq. (3).