Solution to tutorial sheet 3

Discussion topic: What is the scope of the "Glauber model" introduced in the lecture?

7. Nuclear thickness and nuclear overlap functions Part of the solution are on the Mathematica file.

The nuclear density of large spherical nuclei is often approximated by the Woods–Saxon distribution

$$\rho_A(\vec{r}) = \frac{\rho_0}{1+\mathrm{e}^{\frac{|\vec{r}|-R}{a}}} \tag{1}$$

where the values of ρ_0, R and a depend on the nucleus under consideration. For instance, $R_{\mathrm{Pb}} = 6.68$ fm and $a_{\mathrm{Pb}} = 0.546$ fm for lead or $R_{\mathrm{Au}} = 6.38$ fm and $a_{\mathrm{Au}} = 0.535$ fm for gold — the value of ρ_0, which ensures that ρ_A is normalized to A, is irrelevant in the following.

In the lecture, the nuclear thickness function was defined as ($\mathbf{s}$ denotes a transverse vector)

$$T_A(\mathbf{s}) \equiv \int \frac{\rho_A(\mathbf{s}, z)}{A}\,\mathrm{d}z \tag{2}$$

and the nuclear overlap function (for identical nuclei) at impact parameter $\mathbf{b}$ as

$$T_{AA}(\mathbf{b}) \equiv \int T_A\left(\mathbf{s}-\frac{\mathbf{b}}{2}\right) T_A\left(\mathbf{s}+\frac{\mathbf{b}}{2}\right) \mathrm{d}^2\mathbf{s} \tag{3}$$

i. Using the approach of your choice,[1] compute and plot (contour/density/3D plot? your choice!) the thickness function $T_A(\mathbf{s} = x, y)$ of Pb for $|x|, |y| \leq 10$ fm. Same question for the (nameless) function[2] $T_A(\mathbf{s}-\mathbf{b}/2)T_A(\mathbf{s}+\mathbf{b}/2)$ for the four values $|\mathbf{b}| = 0, 4, 8$ and 12 fm and in the same (x, y)-range.

Solution:

We first compute the value of ρ_0 which can be obtained by the normalization of the density as

$$\rho_0 = \frac{A}{\int \frac{1}{1+e^{(r-R)/a}} r^2 \sin\theta d\theta d\phi} = 0.16 fm^{-3}$$

This is precisely the value of the nuclear density given in any textbook which is the density of nucleons per unit of volume.

The nuclear thickness function can not be computed analytically for a W.S. distribution. Therefore we use Mathematica to do the work for us.

[1] Mathematica, C/C++, Python...

[2] A "natural" notation for this function would be $\mathrm{d}^2T_{AA}(\mathbf{b})/\mathrm{d}^2\mathbf{s}$, cf. Eq. (3).

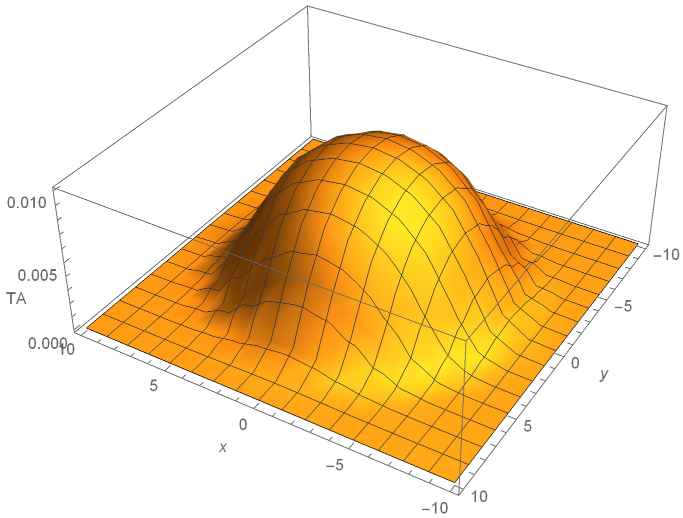

Figure 1: Thickness function of a lead nucleus.

The "nameless" function, which is the integrand of the nuclear overlap function, depends on the impact parameter. Notice that, since the reduced thickness function is the same along the x or y axis we can choose the impact paramter to be in any direction we want. We take $\vec{b} = (b, 0)$ i.e. along the x direction. This is actually how usually he x axis is defined as being the one that is in the direction of the impact parameter. However, you can choose any direction or use polar coordinates.

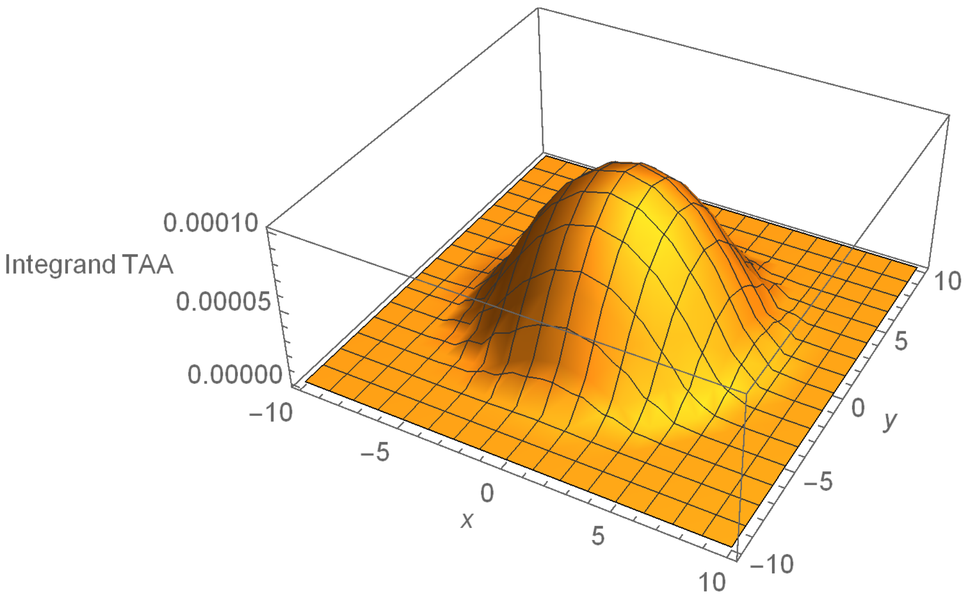

Figure 2: Integrand of the nuclear overlap function.

We can see that for small impact parameters the integrand is peaked at the center and the peaks moves as the impact parameter increases. You can understand that as the probability of two nucleons (one from each nucleus) of having the same position.

ii. In the “Glauber model”, the average number of binary nucleon–nucleon collisions is related to the overlap function by $\langle N_{\rm coll}^{AA}(\mathbf{b})\rangle = A^2 T_{AA}(\mathbf{b})\sigma_{\rm NN}^{\rm inel}$. Compute and plot $\langle N_{\rm coll}^{AA}\rangle$ as a function of $b = |\mathbf{b}|$ in Pb–Pb collisions for impact parameters $0 \le b \le 20$ fm, using $\sigma_{\rm NN}^{\rm inel} = 64$ mb.

Solution: Since A and σ_{NN}^{inel} are constants, the average number of collisions is

$$< N_{coll}^{AA} >= A^2\sigma_{NN}^{inel}T_{AA} = A^2\sigma_{NN}^{inel}\int T_A\left(\mathbf{s}-\frac{\mathbf{b}}{2}\right)T_A\left(\mathbf{s}+\frac{\mathbf{b}}{2}\right)\mathrm{d}^2\mathbf{s}$$

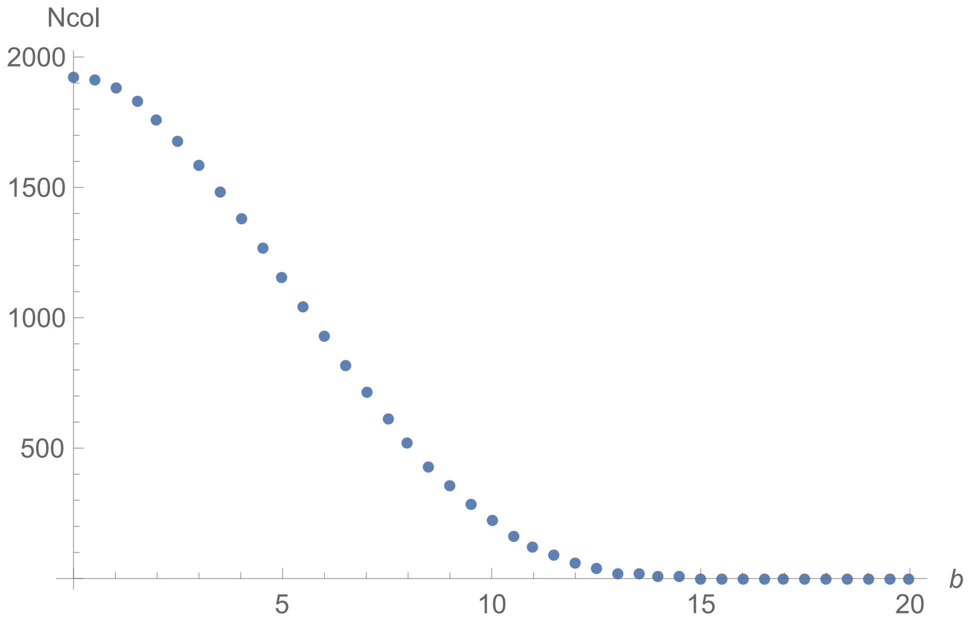

Figure 3: Number of collisions for a lead-lead collision using different impact parameters.

iii. Let $\mathfrak{S}_{AA}(\mathbf{b}) \equiv 1 - \mathrm{e}^{-A^2 T_{AA}(\mathbf{b})\sigma_{\rm NN}^{\rm inel}}$. Compute and plot for $0 \le b \le 20$ fm the function

$$\frac{\mathrm{d}\sigma_{AA}^{\rm inel}(b)}{\mathrm{d}b} \equiv \int \mathfrak{S}_{AA}(\mathbf{b})\, b\, \mathrm{d}\varphi_{\mathbf{b}}, \tag{4}$$

where $\varphi_{\mathbf{b}}$ denotes the azimuthal angle of $\mathbf{b}$, for Pb–Pb collisions, using the same value of $\sigma_{\rm NN}^{\rm inel}$ as in **ii.** Eventually, give the value of the integral

$$\int_0^\infty \frac{\mathrm{d}\sigma_{AA}^{\rm inel}(b)}{\mathrm{d}b}\,\mathrm{d}b, \tag{5}$$

where in practice you may take 20 fm as upper bound.

Solution: The differential cross section will be

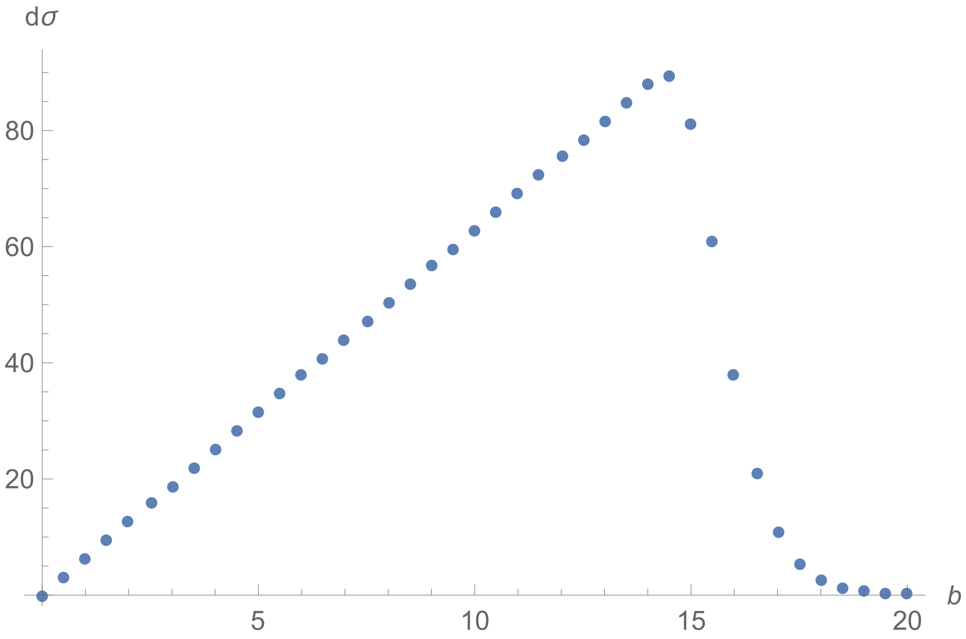

Figure 4: Differential cross section (fm) as function of the impact parameter (fm).

And, if we integrate the impact parameter dependence we obtain

$$\int_0^{20} \frac{\mathrm{d}\sigma_{AA}^{\mathrm{inel}}(b)}{\mathrm{d}b}\,\mathrm{d}b = 792.9 fm^2 \tag{6}$$

For comparison, if the two nuclei were perfect spheres with a fixed radius $R_{Pb} = 6.68 fm$ the total cross section would be the area of a circle of $R = 2R_{Pb}$ which is 560 fm^2 which is comparable to what we obtained before.

Can you explain why we obtain $\sigma = 792.9 fm^2$ in one case and $\sigma = 560 fm^2$ for perfect spheres?

If you have automatized the calculations of this exercise, you may repeat everything for other sets of parameters — and possibly another form for ρ_A — if some day you want/need to.