Tutorial sheet 2

Discussion topic: What are the center-of-mass energy per nucleon pair, the longitudinal rapidity, the pseudorapidity, the transverse mass?

$c = 1$ throughout the exercise sheet.

4. Rapidity of the nuclei

At the Large Hadron Collider, there have been Pb–Pb collisions at $\sqrt{s_{\mathrm{NN}}} = 2.76\,\mathrm{TeV}$ and at 5.02 TeV. What is the rapidity y_{Pb} of the Pb beams in those collisions?
More generally, use your favorite plotting software to plot y_{Pb} vs. $\sqrt{s_{\mathrm{NN}}}$ for $\sqrt{s_{\mathrm{NN}}} \leq 10$ TeV.

Hint: In case you need it, the mass of a lead nucleus was given in exercise **1.**

5. Center-of-momentum energy and rapidity per nucleon pair

Again at the LHC, the beams of nuclei — here denoted A_1 and A_2 — circulating in opposite directions in the accelerator have the same "rigidity" $|\vec{p}|/q$ with $\vec{p}$ and q the (total) momentum and electric charge of the nucleus. (Bonus question: can you see why?)

i. Show that the center-of-mass energy per nucleon pair for an ultrarelativistic $A_1 + A_2$ collision is given by

$$\sqrt{s_{\mathrm{NN}}} \simeq 2|\vec{p}_p|\sqrt{\frac{Z_1 Z_2}{A_1 A_2}} \tag{1}$$

where Z_1, Z_2 are the proton numbers of the colliding nuclei and $|\vec{p}_p|$ the momentum in the lab frame of a proton with the same rigidity as the nuclei.
What is the value of $\sqrt{s_{\mathrm{NN}}}$ for p–Pb collisions with protons of energy 6.5 TeV?

ii. Show that the corresponding center-of-momentum frame of nucleon–nucleon collisions has the rapidity

$$|y_{\mathrm{NN}}| = \frac{1}{2}\ln\frac{Z_1 A_2}{A_1 Z_2} \tag{2}$$

in the lab frame. What value does this give for p–Pb collisions at the LHC?

Note that the c.m. frame of nucleon–nucleon collisions considered here actually differs from the true c.m. frame of the $A_1 + A_2$ collision, whose rapidity can be computed with the help of the following exercise.

6. Center-of-momentum rapidity in asymmetric collisions

Consider the collision $a + b \to \cdots$ where a and b have respective masses m_a, m_b and rapidities (in the lab frame) y_a, y_b. Show that the rapidity of the center-of-momentum frame of the system in the lab frame is

$$|y_{\mathrm{c.m.}}| = \frac{y_a + y_b}{2} + \frac{1}{2}\ln\frac{m_a \mathrm{e}^{y_a} + m_b \mathrm{e}^{y_b}}{m_a \mathrm{e}^{y_b} + m_b \mathrm{e}^{y_a}}. \tag{3}$$