Tutorial sheet 10

Discussion topic: Femtoscopy: what are "HBT radii"? How are they extracted from experimental data? How are they affected by the expansion of the fireball?

Throughout the exercise sheet, a system of units such that the constants c and $\hbar$ equal 1 is used.

20. A toy model for the emission function

Consider a model with the emission function

$$S(\mathsf{x},\mathsf{p}) = \frac{NE_{\vec{p}}}{(2\pi R_s P_0)^3}\exp\left[-\frac{1}{2(1-s^2)}\left(\frac{\vec{r}^2}{R_0^2} - 2s\frac{\vec{r}\cdot\vec{p}}{R_0 P_0} + \frac{\vec{p}^2}{P_0^2}\right)\right]\delta(t - t_{\text{f.o.}}) \tag{1}$$

where N, R_0, P_0, $0 \leq s < 1$, and $t_{\text{f.o.}}$ are parameters while $R_s \equiv R_0\sqrt{1-s^2}$. On the left hand side, we have used the notations $\mathsf{x} \equiv (t,\vec{r})$, $\mathsf{p} \equiv (E_{\vec{p}},\vec{p})$.

Suggest an interpretation for the parameters. Compute the two-particle correlation function $\mathcal{C}(\vec{K},\vec{q})$ using the two formulas

$$\mathcal{C}(\vec{K},\vec{q}) = 1 + \frac{\left|\int S(\mathsf{x},\mathsf{K})\,\mathrm{e}^{\mathrm{i}\mathsf{q}\cdot\mathsf{x}}\,\mathrm{d}^4\mathsf{x}\right|^2}{\int S\left(\mathsf{x},\mathsf{K}+\frac{\mathsf{q}}{2}\right)\mathrm{d}^4\mathsf{x}\int S\left(\mathsf{x},\mathsf{K}-\frac{\mathsf{q}}{2}\right)\mathrm{d}^4\mathsf{x}} \quad \text{and} \quad \mathcal{C}(\vec{K},\vec{q}) = 1 + \frac{\left|\int S(\mathsf{x},\mathsf{K})\,\mathrm{e}^{\mathrm{i}\mathsf{q}\cdot\mathsf{x}}\,\mathrm{d}^4\mathsf{x}\right|^2}{\left[\int S(\mathsf{x},\mathsf{K})\,\mathrm{d}^4\mathsf{x}\right]^2} \tag{2}$$

and compare the results.

21. Bose–Einstein statistics and pair correlation functions

Consider pions described by plane waves of the form $\psi = \mathcal{N}\,\mathrm{e}^{\mathrm{i}\mathsf{p}\cdot\mathsf{x}}$ where $\mathcal{N}$ is a "normalization" constant that plays no role in the following.

i. Write down (symmetrization!) a normalized wave function Ψ_{12} for a system of two pions with four-momenta p_1, p_2 and positions x_1, x_2. Express Ψ_{12} in terms of $\mathsf{K} \equiv \frac{1}{2}(\mathsf{p}_1 + \mathsf{p}_2)$ and $\mathsf{q} \equiv \mathsf{p}_1 - \mathsf{p}_2$.

ii. Defining single- and pair-momentum probabilities by

$$\mathcal{P}_1(\mathsf{p}) \equiv \int \rho(\mathsf{x})|\psi|^2\,\mathrm{d}^4\mathsf{x} \quad , \quad \mathcal{P}_2(\mathsf{p}_1,\mathsf{p}_2) \equiv \int \rho(\mathsf{x}_1)\rho(\mathsf{x}_2)\left|\Psi_{12}\right|^2\mathrm{d}^4\mathsf{x}_1\,\mathrm{d}^4\mathsf{x}_2, \tag{3}$$

compute the pair correlation function $\mathcal{C}(\vec{K},\vec{q}) \equiv \mathcal{P}_2(\mathsf{p}_1,\mathsf{p}_2)/\left[\mathcal{P}_1(\mathsf{p}_1)\mathcal{P}_1(\mathsf{p}_2)\right]$.