Tutorial sheet 1

Discussion topic: In which sense are quarks "free" in a quark-gluon plasma? How does this relate to QCD?

1. Collision kinematics

In a heavy-ion collision at a collider, the nuclei that collide move (along the z-axis) with velocities $+v$ and $-v$ with respect to the "laboratory frame" in which the detectors are at rest. For simplicity, a nucleus will be assimilated to a sphere of radius R.

i. What is the size ("diameter") of one of the nuclei along the z-axis from the point of view of an observer at rest in the laboratory?
When the nuclei collide, they fly through each other without being decelerated — to some extent, they are "transparent". How long does it take in the laboratory frame for the two nuclei to cross each other?

ii. At the Large Hadron Collider, lead nuclei — (rest) mass $m_{\mathrm{Pb}} \simeq 194$ GeV, "radius" $R_{\mathrm{Pb}} \simeq 6.6$ fm — are accelerated to an energy $E \approx 522$ TeV each. How long does it take for the two Pb nuclei to cross each other? You may express your answer either in fm/c or in yoctoseconds (1 ys $= 10^{-24}$ s). Why are the colliding nuclei often referred to as "pancakes"?

2. A useful set of coordinates

To describe the system created in a heavy ion collision, especially at high energies, it is customary to replace the time t and the "longitudinal" spatial coordinate z along the flight direction of the colliding nuclei by other, equivalent coordinates: the "proper time" τ and "spatial rapidity"[1] ς

$$\tau \equiv \sqrt{t^2 - z^2} \quad , \quad \varsigma \equiv \frac{1}{2}\log\frac{t+z}{t-z} \text{ where } |z| < t, \tag{1}$$

in a system of units in which $c = 1$. Implicit in this definition is that the nuclei met at $z = 0$ at time $t = 0$.

i. Check that the relations defining τ and ς can be inverted, yielding[2]

$$t = \tau\cosh\varsigma \quad , \quad z = \tau\sinh\varsigma. \tag{2}$$

(Hint: $\frac{1}{2}\log\frac{1+u}{1-u}$ has a simpler expression in terms of hyperbolic functions.)

ii. Draw on a space-time diagram — with t on the vertical axis and z on the horizontal axis — the lines of constant τ and those of constant ς.

iii. What is the motion of an observer $\mathcal{O}_\varsigma$ at a constant finite ς with respect to (hereafter abbreviated: w.r.t.) an observer $\mathcal{O}_0$ at rest at $z = 0$? In particular, how is ς related to the velocity of $\mathcal{O}_\varsigma$ w.r.t. $\mathcal{O}_0$?

Consider now a third observer $\mathcal{O}_{\varsigma'}$ at spatial rapidity ς' w.r.t. $\mathcal{O}_0$. Can you guess (no calculation!) what is the spatial rapidity of $\mathcal{O}_{\varsigma'}$ w.r.t. $\mathcal{O}_\varsigma$ — that is, the spatial rapidity of $\mathcal{O}_{\varsigma'}$ you would obtain by using Lorentz coordinates comoving with $\mathcal{O}_\varsigma$, and thus properly boosted along the z-direction w.r.t. the original (t, z)-coordinates of $\mathcal{O}_0$?

3. A problem of collider physics

This exercise should help you appreciate one of the difficulties faced by the people tuning the behavior of a collider to ensure that two beams of particles circulating in opposite directions meet at the desired point, inside a detector.

[1] This is the LaTeX character `\varsigma`
[2] Note the analogy with polar coordinates (r, θ)!

Put your forearms parallel to each other, about 50 cm away from each other. Turn your hands inwards, and point your left and right forefingers — that's their job! — towards each other. Close your eyes (after reading the sentence till the end!) and move your hands inwards: how many attempts will you need until the tips of your forefingers meet "head on"?

At the LHC, the proton beams at the interaction points have a sub-millimeter transverse size…