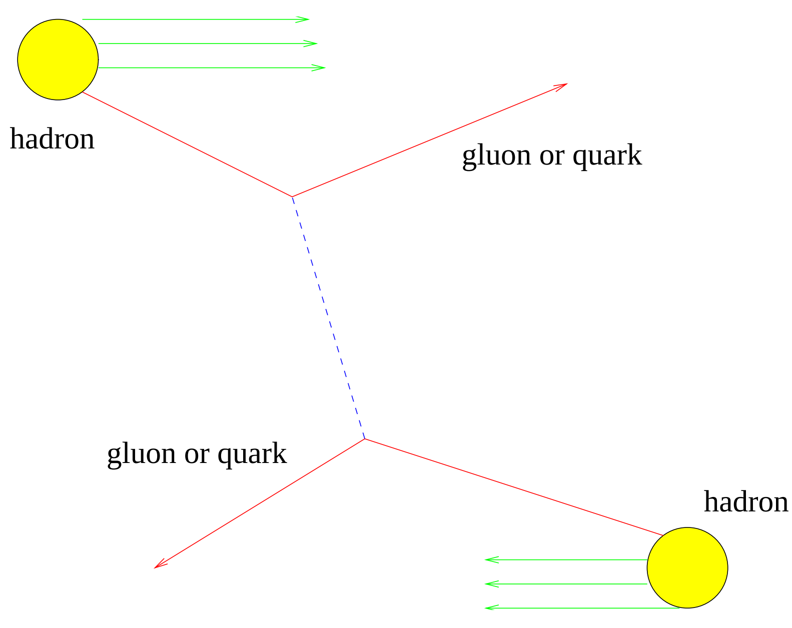
High Energy Hadron Hadron Collisions
and
Initial Conditions for the QGP

Hope: High energy collisions are controlled by hard processes where the coupling is weak.

Problem:

$$\frac{d\sigma}{dy d^2 p_T} \sim \frac{\alpha_s^2}{p_T^4}$$

blows up at small p_T.



Hard Processes:

The total cross section is infrared sensitive:

$$\frac{d\sigma}{dy} \sim \alpha_s^2 \int_{\Lambda_{QCD}} \frac{d^2p_T}{p_T^4}$$

Will argue cutoff at small p_T is Q_{sat}:

$$\frac{1}{\pi R^2}\frac{dN}{d^2p_T dy} = \kappa \frac{1}{\alpha_s} \frac{1}{ln(Q_{sat}^2/\Lambda_{QCD}^2)} \frac{Q_{sat}^4}{p_T^4}$$

is cutoff in the infrared.

If:

Physics is classical fields => then scale invariance =>

$$\frac{1}{\pi R^2}\frac{dN}{d^2p_T dy} = \frac{1}{\alpha_s} F(Q_{sat}^2/p_T^2)$$

where

$F \sim Q_{sat}^4/p_T^4$ for large $p_T >> Q_{sat}$
and $F \sim constant$ for $p_T << Q_{sat}$.

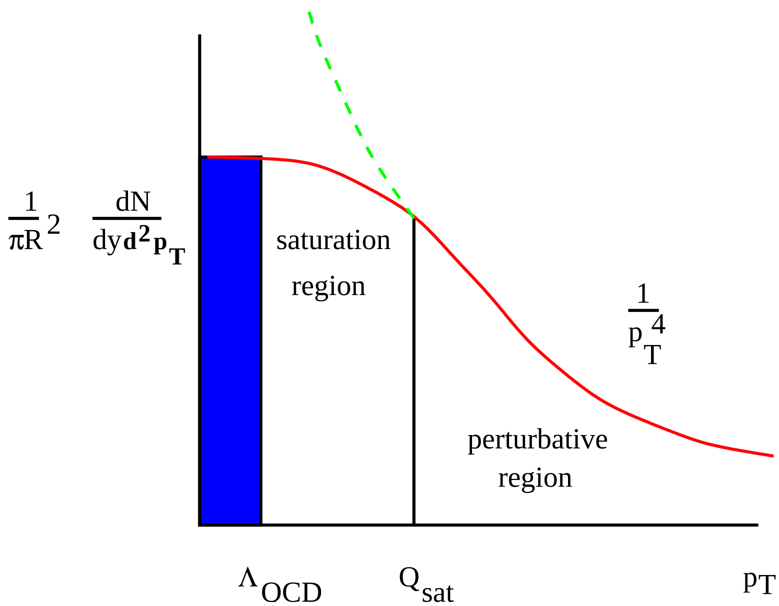
Recall that $Q_{sat}^2 \sim A^{1/3}$

$$\frac{dN}{d^2p_T dy} \sim \pi R^2 \ \frac{Q_{sat}^4}{p_T^4} \sim A^{4/3}/p_T^4$$

at large p_T. At small p_T, $\sim \pi R^2 \sim A^{2/3}$.

$$\int d^2p_T \frac{dN}{d^2p_T dy} \sim \pi R^2 Q_{sat}^2/\alpha_s \sim A/\alpha_s.$$

Hard Processes and Saturation



What is Q^2_{sat}?

$$Q^2_{sat} = \alpha_s^2 \frac{1}{\pi R^2} Q^2_{color}$$

$$Q^2_{color} = \frac{1}{N_c} \frac{1}{N_c^2 - 1}\ tr \tau_a^2 = \frac{1}{2N_c}$$

for quarks and for gluons is

$$Q^2_{color} = \frac{1}{(N_c^2-1)^2}\ tr T_a^2 = \frac{N_c}{N_c^2-1}$$

or for the sum

$$Q^2_{color} = \frac{N_q}{2N_c} + \frac{N_c N_g}{N_c^2-1}$$

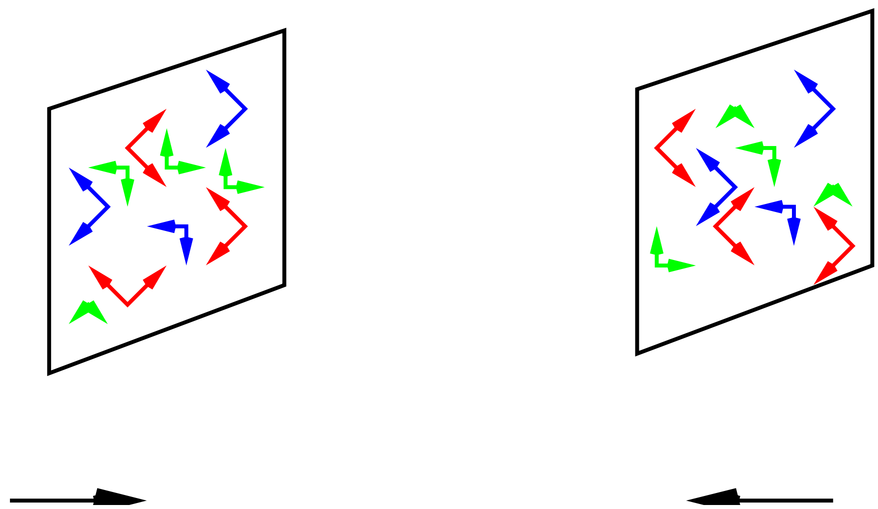
where

$$N_g = \int_{x_0}^1 dx' G(x', Q^2)$$

and

$$N_q = \int_{x_0}^1 dx' \left\{ q(x', Q^2) + \overline{q}(x', Q^2) \right\}$$

Hadron Collisions



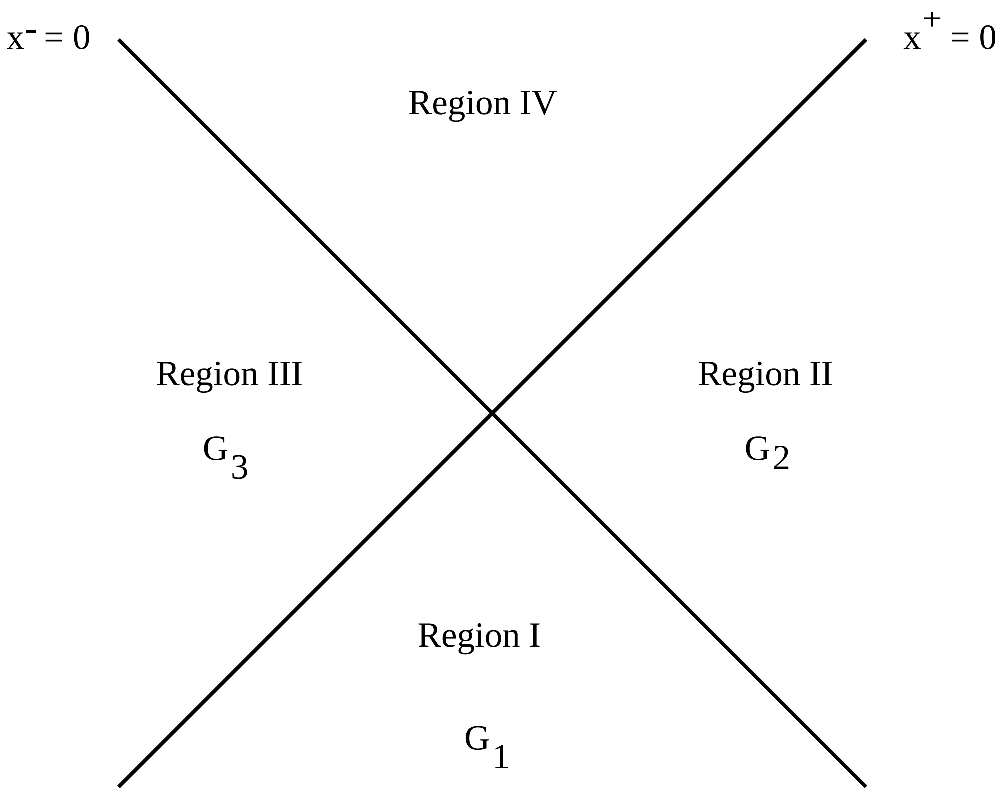
The fields before the collision:

Nucleus 1:$F^{i+} \sim \delta(x^-)$, $F^{ij} \sim 0$, $F^{i-} \sim 0$.
Nucleus 1:$F^{i-} \sim \delta(x^+)$, $F^{ij} \sim 0$, $F^{i+} \sim 0$.

Plane polarized and random color.
Fields are $2-d$ gauge transfoorms of zero field everywhere but in the forward lightcone.

In the forward lightcone matter is produced.
No solution with gauge transform of vacuum in all light cones.

Initial Conditions for QGP



Discontinuities in gauge transformations determine ρ along the backwards light cone. In forward light cone, no gauge transform will determine ρ along both forward cones.

Look for solution independent of

$$\eta = \frac{1}{2} ln \left\{ \frac{t+z}{t-z} \right\}$$

and function only of $\tau = \sqrt{t^2 - z^2}$ and x_T.

Form of the Fields

Before the collision:

$$A^+ = A^- = 0$$

$$\mathsf{A}^i = \theta(x^-)\theta(-x^+)\alpha^i_1(x_T) + \theta(-x^-)\theta(x^+)\alpha^i_2(x_T)$$

After the collision:

$$A^+ = x^+\alpha(\tau, x_T)$$

$$A^- = x^-\beta(\tau, x_T)$$

$$A^i = \alpha^i_3(\tau, x_T)$$

Look for solution in the gauge

$$x^+A^- + x^-A^+ = 0$$

In the forward light cone:

$$\frac{1}{\tau^3}\partial_\tau \tau^3 \partial_\tau \alpha - [D^i, [D_i, \alpha]] = 0$$

$$\frac{1}{\tau}\partial_\tau \tau \partial_\tau \alpha^i_3 - ig\tau^2[\alpha, [D^i, \alpha]] - [D_j, [F^{ji}]] = 0$$

Regularity at $\tau \to 0$:

$$\alpha^i_3(0, x_T) = \alpha^i_1(x_T) + \alpha^i_2(x_T)$$

$$\alpha(0, x_T) = \frac{-ig}{2}[\alpha_{1i}(x_T), \alpha^i_2(x_T)]$$

Asymptotic Solutions:

Large τ

$$\alpha(\tau, x_T) = V\epsilon(\tau, x_T)V^\dagger$$

$$\alpha_3^i(\tau, x_T) = V\left\{\epsilon^i(\tau, x_T) + \frac{i}{g}\partial^i\right\}V^\dagger$$

Fields are gauge rotations of small fluctuations and solve free field equations
Asymptote to:

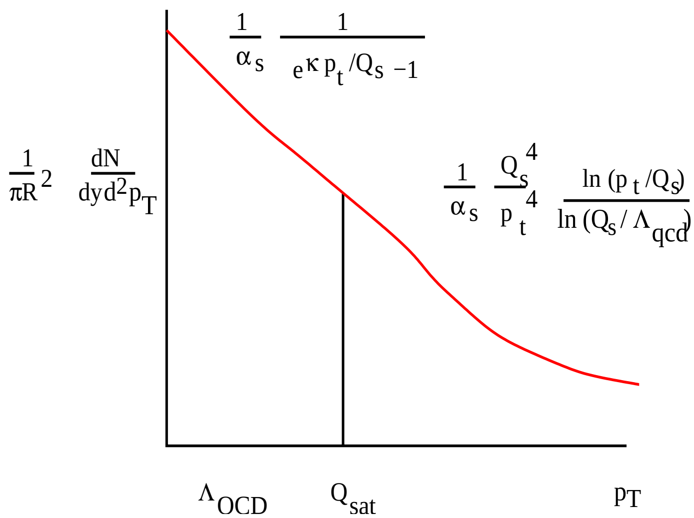
$$\epsilon^a(\tau, x_T) = \int \frac{d^2k_T}{(2\pi)^2\sqrt{2\omega}}\frac{1}{\tau^{3/2}}\left\{a_1^a(k_T)e^{-ikx} + c.c.\right\}$$

$$\epsilon_i^a(\tau, x_T) = \int \frac{d^3k_T}{(2\pi)^3\sqrt{2\omega}}\epsilon_{ij}\frac{k^j}{\omega}\frac{1}{\tau^{1/2}}\left\{a_2^a(k_T)e^{-ikx} + c.c.\right\}$$

where $\omega = \mid k_T \mid$.

Nara, Krasnitz and Venugopalan:
Numerically solve equations for fixed ρ.
Compute gluon production.
Average over ρ with Gaussian weight.
Compute multiplicity distribution of produced gluons, flow patterns etc.

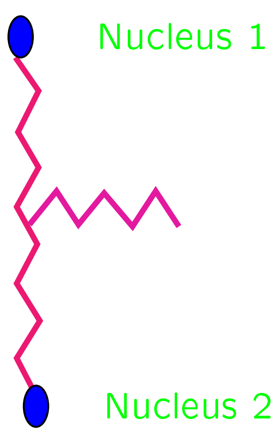
NKV Results:



Similar to Bose-Einstein at small p_T

$1/p_T^4$ at large p_T

The high p_T tail is computable from:

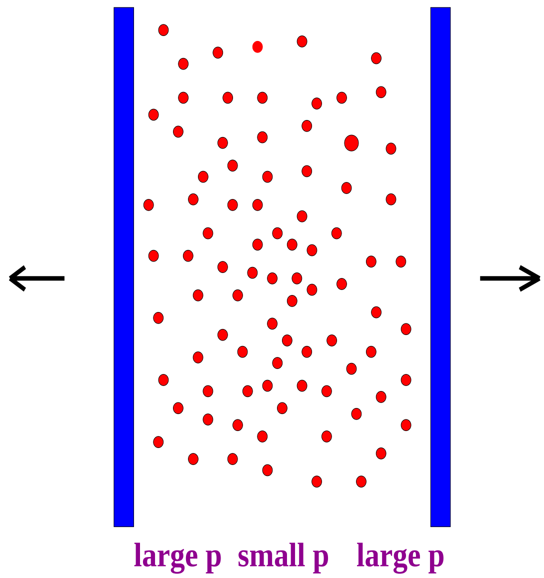


The tail integrates to

$$\frac{1}{ln(Q_{sat}^2/\Lambda_{QCD}^2)}$$

smaller contribution than the piece below Q_{sat}

Thermalization:



Slow particles form first. Fast are last.

Like Hubble expansion.
$V \sim 1/\tau$

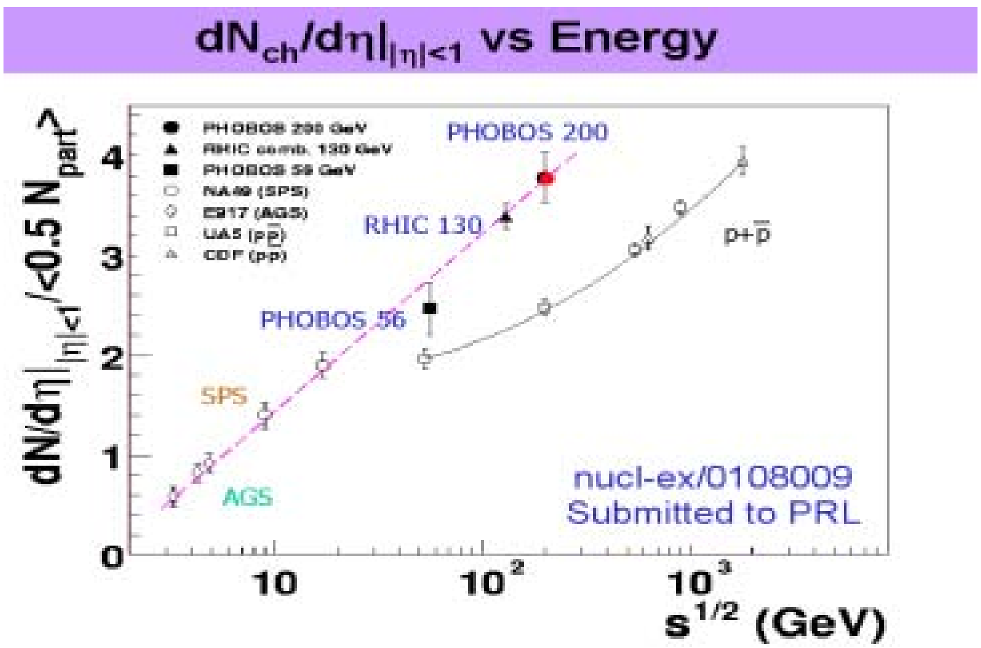
Entropy conserved:
$S \sim T^3 \tau R^2$
$T \sim \tau^{-1/3}$

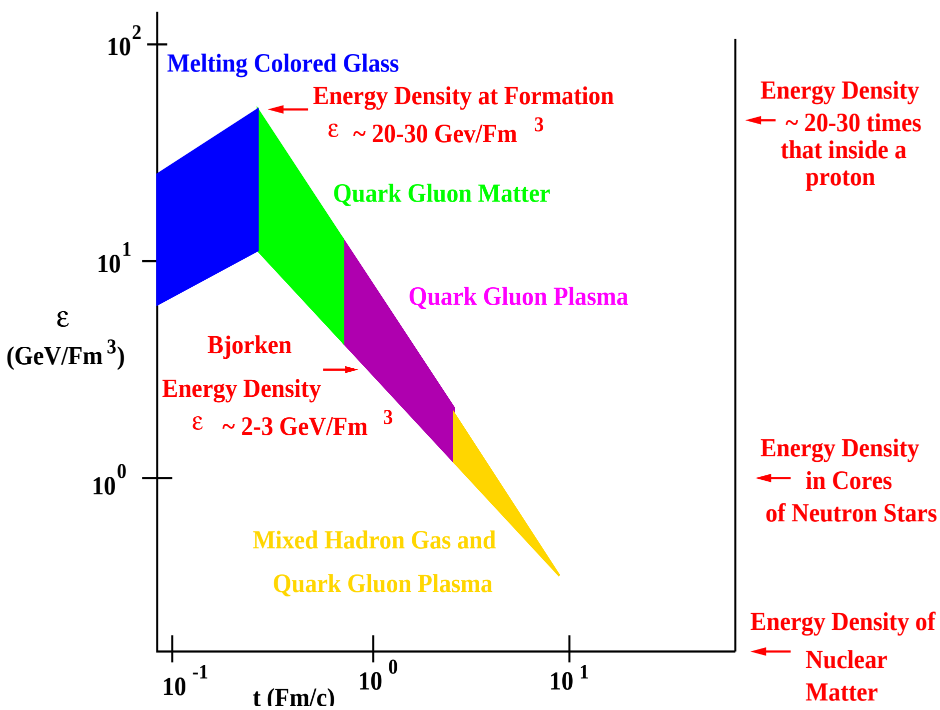
Energy density $\sim T^4$ for thermal $\epsilon \sim \tau^{-4/3}$
For non thermal $\epsilon \sim \tau^{-1}$
At late times when energy per particle is slowly varying measure

$$\frac{dE}{dy} = \epsilon \; \tau \pi R^2$$

Can measure $< m_T >$ and dN/dy.

Multiplicity and Energy Density





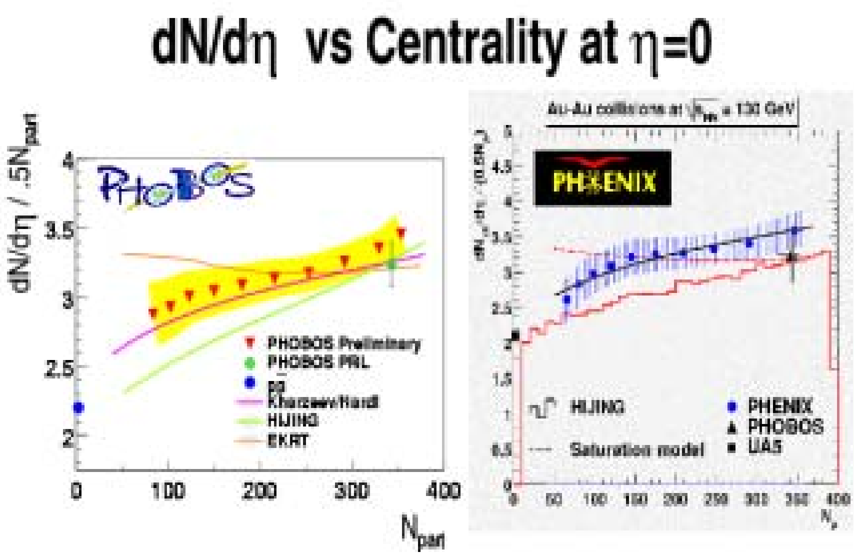
Multiplicity and Centrality

N_{part} are the number of nucleons which scattered.

Increasing N_{part} decreases impact parameter.

$$Q^2_{sat} \sim N^{1/3}_{part}$$

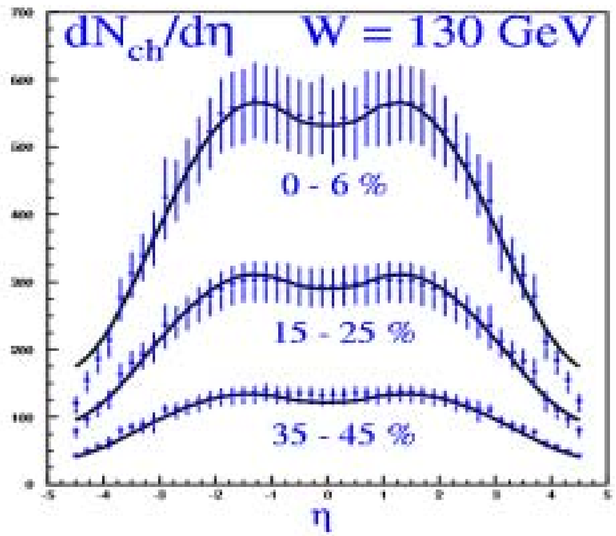
$$N \sim \pi R^2 \; Q^2_{sat}/\alpha_s(Q_{sat}) \sim N_{part}/\alpha_s(Q_{sat})$$

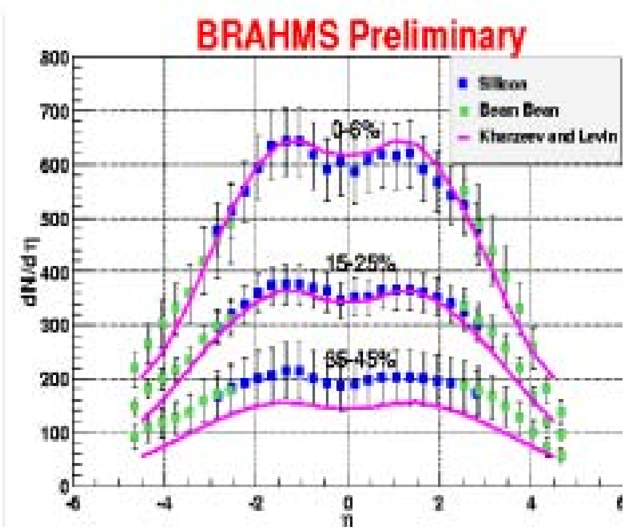


Good agreement with saturation model.

Assumption about N_{part} dependence of Q_{sat}

Rapidity Dependence:



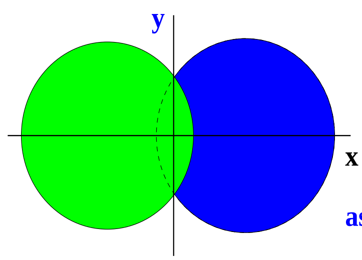


dN/dy measured by Phobos and Brahms.

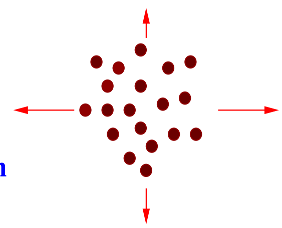
Computed in Color Glass picture by Nardi, Levin and Kharzeev

Will work through pA example later Same technique.

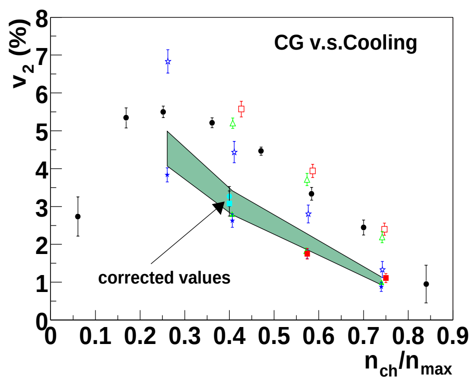
Flow

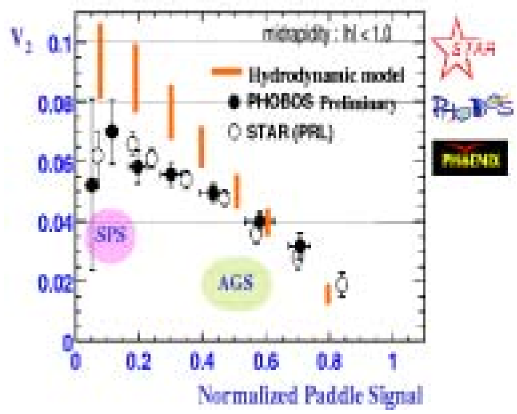


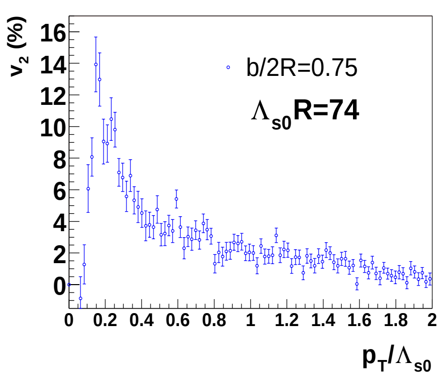
In region of overlap
asymmetry of spatial
distribution ->
asymmetry of momentum
distribution



$v_2 = \langle \cos(2\phi) \rangle$ $\tan(\phi) = Py/p_x$







p_T dependence of Color Glass is too soft.
Needs at least some hydro.

Hydro must contribute some of the effect!

Thermalization:

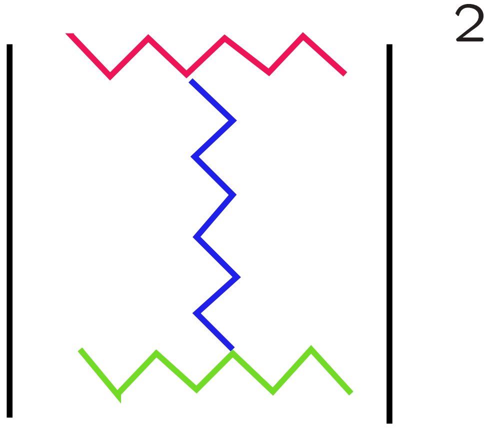
α_s weak $=>$ long time.

When CGC melts: $< p_T >$ freezes,

N/V decreases

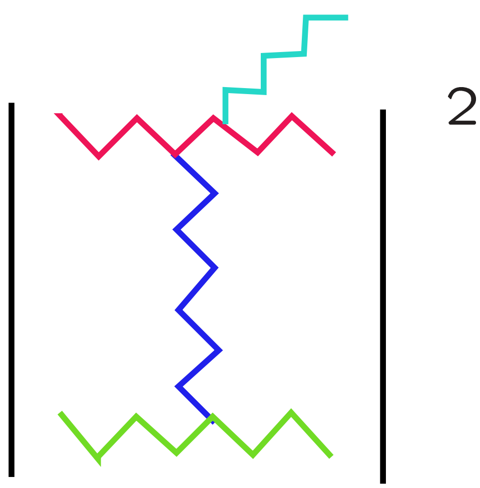
Local restframe $p_z = 0$

Becomes Dilute Bose gas



$\sim \frac{1}{Q_D^2} \to \alpha_s^2 ln(Q_D)$

Forward-backward cancellation



$\sim \frac{1}{Q_D^2}\alpha_s^3 \sim \alpha_s^2$

for $Q_D at$

equilibrium

value.

Can get bigger!

Multiparticle

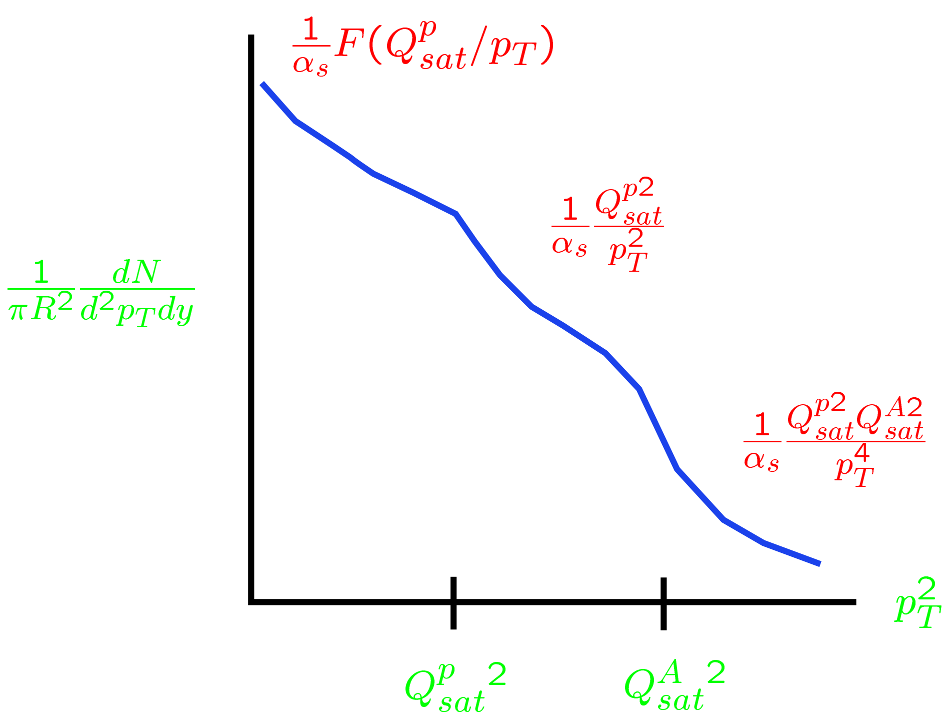
thermalization!

pA Scattering:

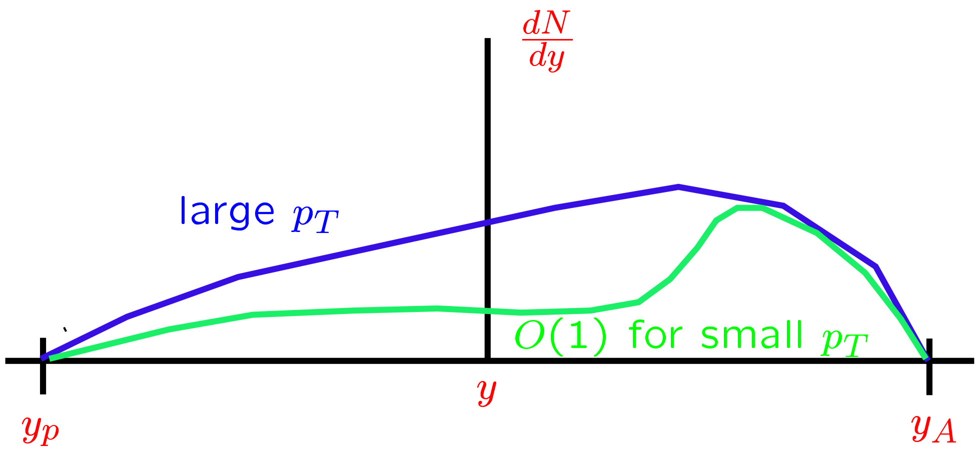
Trick: Solve to all order in ρ_A
First order in ρ_p.

In forward light cone:
Gauge transform of plane waves.
Boundary condtions fixed as before.

Exactly Solvable.



pA Rapidity Distributions



$A'A$ collisions have rich structure.
Also unequal rapidity AA